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Interactive Compromise Programming Approach for Solving Vendor Selection Problems under Fuzziness

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Abstract

This paper studies a Vendor Selection Problem (VSP) with fuzzy parameters in the price of a unit item, an upper limit of the quantity available, and an aggregate demand for the item. These fuzzy parameters are characterized as fuzzy numbers. An extended efficiency concept called that α-efficient solution is introduced using the-level sets of fuzzy numbers. A fuzzy programming approach is applied by defining a membership function after converting the fuzzy VSP into an equivalent deterministic VSP. A linear membership function is being used to obtain $\alpha-$optimal compromise solution. An interactive procedure for obtaining α-optimal compromise solution is also presented. An illustrative numerical example is given to clarify the obtained results.

Keywords: Optimization, Multi-objective integer programming, Vendor selection problem, Decision-making, Fuzzy parameters, Triangular fuzzy numbers, α-level sets, α-efficient solution, Linear membership function, Fuzzy programming, Interactive procedure, Supply chain management, α-optimal compromise solution.

1 | Introduction

In a supply chain, vendor selection includes the selection of the right vendors and their quota allocation, which also needs to consider a variety of vendor attributes such as price and quality. A Vendor Selection Problem (VSP) must consider these attributes because of their direct impact on final product dimensions, such as cost and quality. Vendor selection decisions play an important role in supply chain management and significantly impact a firm's competitiveness because purchases from vendors account for a large percentage of the total cost for many firms.

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Vendor selection has long been regarded as one of the most critical functions performed by the purchasing department. Several methods have been proposed for solving VSPs in deterministic or stochastic environments. The linear weighting method proposed by Wind and Robinson [1] for vendor selection decisions is one of the most common ways for rating different vendors on performance criteria for quota allocation. Linear Programming (LP), Mixed Integer Programming (MIP), and Multi-Objective Programming (MOP) are also commonly used techniques. Pan [2] developed a single-item LP model to minimize the aggregate price for quality, service level, and lead time constraints. Bender et al. [3] proposed an MIP. They used IBM to select vendors and order quantities to minimize purchasing, inventory, and transportation costs, but a specific mathematical formulation is not provided. Weber et al. [4] used the Data Environment Analysis (DEA) as a negotiation tool for a buyer in selecting vendors.

In many scientific areas, such as system analysis and operator research, a model has to be set up using only approximately known data. Fuzzy sets theory, introduced by Zadeh [5], makes this possible. Fuzzy numerical data can be represented by employing fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [6] extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle. Dubois and Prade have studied fuzzy linear constraints with fuzzy numbers [6]. Tanaka and Asai [7] formulated a fuzzy LP problem to obtain a reasonable solution under consideration of the ambiguity of parameters. Rommelfonger et al. [8] presented an interactive method for solving Multi-Objective Linear Programming (MOLP) problems, where coefficients of the objective functions and/or constraints are known exactly but imprecisely. Zhao et al. [9] studied the complete solution set for the most generalized symmetrical fuzzy LP problems in which both fuzzy (non-fuzzy) equality and inequality constraints are included, and both linear or non-linear membership functions are allowed. Fuzzy LP with multiple objective functions was introduced by Zimmermann [10]. Sakawa and Yano [11] introduced the concept of α-Pareto optimality of fuzzy parametric programs. Kumar et al. [12] treated VSPs as fuzzy MIP formulation that incorporates the three crucial goals: cost-minimization, quality maximization, and maximization of on-time-delivery with realistic constraints such as meeting the buyers' demand, vendor's capacity, vendors' quota flexibility, etc. Díaz-Madroñero et al. [13] considered VSP with fuzzy goals. They developed an interactive method for solving multi-objective VSPs where fuzzy data are represented using S-curve membership functions.

Amid et al. [14] and Weber and Current [4] introduced a model of the MOLP problem of supplier selection model. Kumar and Roy [15] developed a rule-based model to evaluate the performance of vendors supplying components and raw materials to a multinational organization engaged in designing, manufacturing, and delivering a range of products covering various stages of electric power transmission and distribution systems. Mendoza et al. [16] designed a new multi-criteria method to solve the general supplier selection problem. He et al. [17] studied a VSP in which the buyer allocates the order quantity for an item among a set of suppliers such that the required aggregate quality, service, and lead time requirements are achieved at minimum cost. Ware et al. [18] provided an extensive state-of-the-art literature review and critique of the studies on various supplier selection problems over the past two decades. Ekhtiari and Poursafary [19] studied the process of selecting the vendors simultaneously in three aspects: multiple criteria, random factors, and reaching efficient solutions with the objective of improvement. Arikan [20] introduced a novel interactive solution for solving a multiple-sourcing supplier selection problem involving three objective functions with fuzzy demand levels and/or fuzzy aspiration levels of objectives. Lai and Hwang [21] introduced a classification of solution approaches for fuzzy multiple-objective decision-making problems. Sakawa [22] introduced the basics of interactive fuzzy multiple-objective optimization. Alves and Climaco [23] reviewed interactive methods for solving multi-objective and mixed integer programming problems. Khalifa [24] proposed an interactive approach combined with the reference direction method and the attainable reference point to solve the multi-objective non-LP problem with fuzzy parameters in the objective function and introduced the first kind of stability set corresponding to the obtained solution. Recently, some articles have studied VSPs [25]–[27].

The rest of the paper is outlined in *Fig. 1*.

Section 2
- Introduces some background information on fuzzy numbers and their level.

Section 3
- Introduces some of assumptions and notions needed in the problem formulation

Section 4
- Formulates VSP in fuzzy environment.

Section 5
- proposes an Interactive fuzzy programming for determing the $\alpha -$ pareto optimal solution .

Section 6
- To demonstrate the suggested algorithm, a numerical example is provided.

Section 7
- The paper is summarized with recommendations for the future.

Fig. 1. Layout of remaining paper.

To discuss our problem conveniently, we shall state some necessary results on interval arithmetic and fuzzy numbers [28], [29].

Let $I(R)=\left\{[a^L, a^U]: a^L, a^U \in R=(-\infty,\infty), a^L \le a^U\right\}$ denote the set of all closed interval numbers on R .

Definition 1. Assume that: $[a^L, a^U], [b^L, b^U] \in I(R)$, we define:

I. $[a^L, a^U](+)[b^L, b^U]=[a^L+b^L, a^U+b^U]\frac{1}{n}.$

II. $[a^L, a^U](-)[b^L, b^U]=[a^L-b^U, a^U-b^L].$

III. $[a^L, a^U](\cdot)[b^L, b^U]=[\min(a.^L b^L, a^L.b^U, a^U.b^L, a^U.b^U),$
$\max(a.^L b^L, a^L.b^U, a^U.b^L, a^U.b^U)].$

IV. The order relation $"\le"$ in $I(R)$ is defined by $[a^L, a^U](\le)[b^L, b^U]$ if and only if $a^L \le b^L, a^U \le b^U$.

Definition 2. Let R be the set of real numbers; the fuzzy number $\tilde{p}$ is a mapping $\mu_{\tilde{a}}: R \to [0, 1]$, with the following properties:

I. $\mu_{\tilde{p}}(x)$ is an upper semi-continuous membership function.

II. $\tilde{p}$ is a convex set, i.e., $\mu_{\tilde{p}}(\lambda x^1+(1-\lambda)x^2) \ge \min\{\mu_{\tilde{p}}(x^1), \mu_{\tilde{p}}(x^2)\}$, for all $x^1, x^2 \in R, 0 \le \lambda \le 1$.

III. $\tilde{p}$ is normal, i.e., $\exists x_0 \in R$ for which $\mu_{\tilde{p}}(x)=1$.

IV. $\text{Supp}(\tilde{p})=\{x: \mu_{\tilde{p}}(x)>0\}$ is the support of a fuzzy set $\tilde{p}$.

Let $F_0(R)$ denote the set of all compact fuzzy numbers on R, that is, for any $g \in F_0(R), g$ satisfies the following:

I. $\exists x \in R: g(x)=1.$

II. For any $0<\alpha \le 1, g_\alpha=[g_\alpha^L, g_\alpha^U]$ is a closed interval number on R.

It is noted that $R \subset I(R) \subset F_0(R)$.

Definition 3. The $\alpha-$level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level α:

$L_\alpha(\tilde{a}) = \{a : \mu_{\tilde{a}}(a) \geq \alpha\}$.

Definition 4. A Triangular Fuzzy Number (TFN) can be entirely represented by a triplet $\tilde{A} = (a_1, a_2, a_3)$, and its interval of confidence at level α is defined by

$$\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3], \quad \text{for all } 0 < \alpha \leq 1.$$

3 | Assumption and Notation

In this VSP, the following assumptions are made.

Assumptions

I. Only one item is purchased from one vendor.

II. Quality discounts are not taken into consideration.

III. No shortage of the item is allowed for any of the vendors.

IV. Lead time and demand for the item are constant and known with certainty.

Notation

In this VSP, the following notation can be used:

n: Number of vendors competing for selection.

D: Aggregate demand for the item over a fixed planning period.

s_i : Price of a unit item of the ordered quantity x_i to the vendor.

ℓ_i : Percentage of the late delivered units by the vendor i.

q_i : Percentage of the rejected units delivered by vendor i.

r_i : Vendor rating values for vendor i.

$P = r \times D$: Least total purchasing value that a vendor can have.

f_i : Vendor quota flexibility for vendor i.

$F = f \times D$: Least value of the flexibility in supply quota that a.

B_i : Budget constraint allowed to each vendor.

U_i : The upper limit of the quantity available for vendors.

4 | Problem Formulation and Solution Concept

Consider a VSP with fuzzy parameters both in the price of the item and aggregate demand (F-VSP) corresponding to the VSP introduced by Kumar et al. [12] as

$$\text{Min } \tilde{Z}_1(x,\tilde{s}) = \sum_{i=1}^{n} \tilde{s}_i(x_i),$$
$$\text{Min } Z_2(x) = \sum_{i=1}^{n} q_i(x_i),$$
$$\text{Min } Z_3(x) = \sum_{i=1}^{n} l_i(x_i),$$

subject to

$$\sum_{i=1}^{n} x_i = \tilde{D},$$
$$x_i \le \tilde{U}_i, \qquad i = 1, 2, ..., n,$$
$$\sum_{i=1}^{n} r_i(x_i) \ge \tilde{P},$$
$$\sum_{i=1}^{n} f_i(x_i) \le \tilde{F},$$
$$\tilde{s}_i(x_i) \le B_i, i = 1, 2, ..., n,$$
$$x_i \ge 0,\ i = 1, 2, ..., n \text{ and integer.}$$

Definition 5 (Fuzzy efficient solution). A point $x^* \in X(\tilde{s}, \tilde{U}, \tilde{D})$ is said to be a fuzzy efficient solution to the F-VSP if and only if there does not exist another $x \in X(\tilde{s}, \tilde{U}, \tilde{D})$, such that: $Z_1(x,p) \le Z_1(x^*, p^*), Z_2(x) \le Z_2(x^*)$ and $Z_3(x) \le Z_3(x^*)$ and $\tilde{Z}_1(x,s) \ne \tilde{Z}_1(x^*, \tilde{s})$ or $Z_2(x) \ne Z_2(x)$ or $Z_3(x) \ne Z_3(x)$.

Assuming that these fuzzy parameters $\tilde{s}_i\,(i = 1, 2, ..., n)$, $\tilde{U}_i\,(i = 1, 2, ..., n)$ and $\tilde{D}$ are characterized by fuzzy numbers [6], let the corresponding membership functions be $\mu_{\tilde{s}_i}(s_i), i = 1, 2, ..., n$; $\mu_{\tilde{U}_i}(U_i), i = 1, 2, ..., n;$ and $\mu_{\tilde{D}}(D)$. We introduce the α-level set of the fuzzy numbers $\tilde{s}$, $\tilde{U}$ and $\tilde{D}$ defined as the ordinary set $(\tilde{s}, \tilde{U}, \tilde{D})_\alpha$ in which the degree of their membership functions exceeds level α.

$$(\tilde{s}, \tilde{U}, \tilde{D})_\alpha = \{(s, U, D) : \mu_{\tilde{s}_i}(s_i) \ge \alpha, i = 1, 2, ..., n; \mu_{U_i}(U_i), i = 1, 2, ..., n; \mu_{\tilde{D}}(D) \ge \alpha\}.$$

For a certain degree of α, the F-VSP can be written in the following non-fuzzy form [11]:

$$\text{Min } Z_1(x,p) = \sum_{i=1}^{n} s_i(x_i) \in \sum_{i=1}^{n} (\tilde{s}_i)_\alpha(x_i),$$
$$\text{Min } Z_2(x) = \sum_{i=1}^{n} q_i(x_i),$$
$$\text{Min } Z_3(x) = \sum_{i=1}^{n} l_i(x_i),$$

subject to

$$\sum_{i=1}^{n} x_i = D \in (\tilde{D})_\alpha,$$
$$x_i \le U_i \in (\tilde{U}_i)_\alpha, \quad i = 1, 2, ..., n,$$
$$\sum_{i=1}^{n} r_i(x_i) \ge \tilde{P},$$
$$\sum_{i=1}^{n} f_i(x_i) \ge \tilde{F},$$
$$s_i(x_i) \in (\tilde{s}_i)_\alpha(x_i) \le B_i, \quad i = 1, 2, ..., n,$$
$$0 < \alpha \le 1, \quad x_j \ge 0, \quad j = 1, 2, ..., n, \text{ and integers.}$$

In the following, for simplicity, we denote the feasible region satisfying the constraints of the α-VSP concerning x by $X(s,U,D)$. It should be emphasized here that in the α-VSP, the parameters (s,U,D) are treated as decision variables rather than constants.

Based on the α-level sets of the fuzzy numbers, we can introduce the concept of an α-efficient solution to the α-VSP as a natural extension of the efficient solution concept.

Definition 6 (α-efficient solution). A point $x^* \in X(s^*, U^* D^*)$ is said to be an α-efficient solution to the α-VSP if and only if there does not exist another $x \in X(s,U,D)$, $(s,U,D) \in (\tilde{s}, \tilde{U}, \tilde{D})_\alpha$ such that: $Z_1(x,p) \le Z_1(x^*, p^*), Z_2(x) \le Z_2(x^*)$, and $Z_3(x) \le Z_3(x^*)$, and $Z_1(x,s) \ne Z_1(x^*, s^*)$ or $Z_2(x) \ne Z_2(x)$ or $Z_3(x) \ne Z_3(x)$, where the corresponding values of parameters (s^*, U^*, D^*) are called $\alpha-$level optimal parameters.

5 | Interactive Fuzzy Programming for Solving the Problem

Bellman and Zadeh [30] introduced three basic concepts: fuzzy goal (G), fuzzy constraints (C), and fuzzy decision (D), and explored the applications of these concepts to decision-making under fuzziness. Their fuzzy decision is defined as follows:

$$D = G \cap C. \tag{1}$$

The membership function characterizes this problem

$$\mu_D(x) = \min(\mu_G(x), \mu_C(x)). \tag{2}$$

To define the membership function of the (α-VSP), let us follow:

Calculate the individual minimum at $\alpha = 0$ as

$$Z_1^{\min} = Z_1(x^0, s^0) = \min\{Z_1(x,s) : x \in X(s,U,D), (s,U,D) \in (\tilde{s}, \tilde{U}, \tilde{D})_{\alpha=0}\}. \tag{3}$$

$$Z_j^{\min} = Z_j(x) = \min\{Z_j(x) : x \in X(s,U,D), (s,U,D) \in (\tilde{s}, \tilde{U}, \tilde{D})_{\alpha=0}, j = 2,3\}. \tag{4}$$

and the individual maximum at $\alpha = 1$ as

$$Z_1^{\min} = Z_1(x^0, s^0) = \max\{Z_1(x,s) : x \in X(s,U,D), (s,U,D) \in (\tilde{s}, \tilde{U}, \tilde{D})_{\alpha=1}\}. \tag{5}$$

$$Z_j^{\max} = Z_j(x^0) = \max\{Z_j(x) : x \in X(s,U,D), (s,U,D) \in (\tilde{s}, \tilde{U}, \tilde{D})_{\alpha=1}, j = 2,3\}. \tag{6}$$

Based on the definition of $Z_1^{\min}, Z_1^{\max}, Z_j^{\min}$ $(j = 2,3)$, and $Z_j^{\max}$ $(j = 2,3)$, Biswal [31] gives a membership function of a multi-objective geometric programming problem which can be implemented for the $\alpha-$VSP as follows:

$$\mu_1(Z_1(x,s)) = \begin{cases} 1, & \text{if } Z_1(x,s) \le Z_1^{\min}, \\ \dfrac{Z_1^{\max} - Z_1(x,s)}{Z_1^{\max} - Z_1^{\min}}, & \text{if } Z_1^{\min} \le Z_1(x,s) < Z_1^{\max}, \\ 0, & \text{if } Z_1(x,s) \ge Z_1^{\max}, \end{cases} \tag{7}$$

and

$$\mu_j(Z_j(x)) = \begin{cases} 1, & \text{if } Z_j(x) \le Z_j^{\min}, \\ \dfrac{Z_j^{\max} - Z_j(x)}{Z_j^{\max} - Z_j^{\min}}, & \text{if } Z_j^{\min} \le Z_j(x) < Z_j^{\max}, j = 2,3, \\ 0, & \text{if } Z_j(x) \ge Z_j^{\max}. \end{cases} \tag{8}$$

Following the fuzzy decision of Bellman and Zadeh [30] with the linear *Membership Functions (11)* and *(12)*, a fuzzy programming model to the α-VSP can be written as follows:

$$
\begin{aligned}
&\text{Max } \min\left\{\mu_1\left(Z_1(x,s)\right), \mu_2\left(Z_2\left(x\right)\right), \mu_3\left(Z_3\left(x\right)\right)\right\},\\
&\text{subject to}\\
&\quad \sum_{i=1}^{n} x_i = D,\\
&\quad x_i \le U_i, i = 1, 2, ..., n,\\
&\quad \sum_{i=1}^{n} r_i(x_i) \ge P,\\
&\quad \sum_{i=1}^{n} f_i(x_i) \le F,\\
&\quad s_i(x_i) \le B_i, i = 1, 2, ..., n,\\
&\quad (s, U, D) \in (\tilde{s}, \tilde{U}, \tilde{D})_\alpha, 0 < \alpha \le 1,\\
&\quad x_i \ge 0, i = 1, 2, ..., n, \text{ and integers.}
\end{aligned}
\tag{9}
$$

By introducing an auxiliary variable ν, *Problem (9)* can be transformed into the following problem:

$$
\begin{aligned}
&\underset{x,s,U,D}{\text{Max}}\ \nu,\\
&\text{subject to}\\
&\quad \mu_1(Z_1(x,s)) \ge \nu, \qquad s \in (\tilde{s})_\alpha,\\
&\quad \mu_j(Z_j(x)) \ge \nu, \qquad j = 2, 3,\\
&\quad \sum_{i=1}^{n} x_i = D, D \in (\tilde{D})_\alpha,\\
&\quad x_i \le U_i, i = 1, 2, ..., n,\\
&\quad U_i \in (\tilde{U}_i)_\alpha, i = 1, 2, ..., n,\\
&\quad \sum_{i=1}^{n} r_i(x_i) \ge P,\\
&\quad \sum_{i=1}^{n} f_i(x_i) \le F,\ s_i(x_i) \le B_i, i = 1, 2, ..., n,\\
&\quad (s, U, D) \in (\tilde{s}, \tilde{U}, \tilde{D})_\alpha, 0 \le \nu \le 1;\ 0 \le \alpha \le 1,\\
&\quad x_i \ge 0, i = 1, 2, ..., n, \text{ and integers.}
\end{aligned}
\tag{10}
$$

By solving *Problem (10)*, we obtain a solution to maximize a smaller satisfactory degree for the decision maker. Unfortunately, *Problem (14)* is not an LP problem even if all the membership functions $\mu_1(Z_1(x,s))$, and $\mu_j(Z_j(x)), j = 2, 3,$ is linear. To solve *Problem (10)* by using the LP technique, we introduce the set-valued functions:

$$
T\left(s_i\right) = \left\{\left(x, \nu\right) : \mu_1\left(Z_1(x, s_i\right) \ge \nu, s_i\left(x_i\right) \le B_i, i = 1, 2, ..., n\right\}. \tag{11}
$$

$$
V_i(U_i, D) = \{x : \sum_{i=1}^{n} x_i = D, x_i \le U_i, i = 1, 2, ..., n\}. \tag{12}
$$

Then, it can be verified that the following relations hold for $T(s_i)$ and $V_i(U_i, D)$, when $x_i \ge 0,\ i = 1, 2, ..., n$ [11].

Proposition 1.

I. If $s_i^1 \le s_i^2$, then $T(s_i^1) \supseteq T(s_i^2)$.

II. If $U_i^1 \le U_i^2$, then $V_i(U_i^1, .) \subseteq V_i(U_i^2, .)$.

III. If $D^1 \leq D^2$, then $V_i(.\,,\,D^1) \subseteq V_i(.\,,\,D^2)$.

From the properties of the α-level set of fuzzy numbers $\tilde{s}_i\ (i=1,2,..,n)$, ; $\tilde{U}_i\ (i=1,\ 2,\ ...,\ n)$, and $\tilde{D}$, it should be noted that the feasible denoted, respectively, by the closed intervals $(\tilde{s}_i)_\alpha = [(s_i)_\alpha^L,\ (s_i)_\alpha^U],\ (\tilde{U}_i) = [(U_i)_\alpha^L, (U_i)_\alpha^U], i=1,2,...,n,$ and $(D)_\alpha = [D_\alpha^L,\ D_\alpha^U]$.

Therefore, with *Proposition 1*, we can obtain an α-optimal compromise solution to the *Problem (12)* by solving the following LP problem:

$$
\begin{aligned}
&\underset{x}{\text{Max}}\ \ \nu,\\
&\text{subject to}\\
&\quad \mu_1(Z_1(x,s^L)) \geq \nu,\\
&\quad \mu_j(Z_j(x)) \geq \nu,\ j=2,3,\\
&\quad 0 \leq \nu \leq 1,\\
&\quad \mu_1(Z_1(x,s^L)) \geq \nu,\\
&\quad \mu_j(Z_j(x)) \geq \nu,\ j=2,3;\\
&\quad 0 \leq \nu \leq 1,\\
&\quad \sum_{i=1}^{n} x_i \leq D^U,\\
&\quad x_i \leq U_i^U,\\
&\quad \sum_{i=1}^{n} r_i(x_i) \geq P^L,\\
&\quad \sum_{i=1}^{n} f_i(x_i) \leq F^U,\\
&\quad s_i^L(x_i) \leq B_i, i=1,2,...,n.
\end{aligned}
\qquad (13)
$$

6 | An Interactive Procedure

Step 1. Calculate the individual minimum and maximum of each objective function under the given constraints for $\alpha = 0$, and $\alpha = 1$, respectively.

Step 2. Ask the DM to select the initial value of $\alpha (0 < \alpha \leq 1)$.

Step 3. Elicit the membership functions, $\mu_1(Z_1(x,s))$, and $\mu_1(Z_j(x)), j=2,3$.

Step 4. Formulate *Problems (9)* and *(13)*, and solve them to obtain α-optimal compromise solution.

7 | Numerical Example

Consider the following problem.

Table 1. Vendor source with fuzzy data for the problem.

Vendor No.	1	2	3	4
$\tilde{s}_i$ ($)	(2, 5, 6)	(1, 2, 5)	(4, 7, 9)	(0, 1, 2)
q_i (%)	0.05	0.03	0	0.02
l_i (%)	0.03	0.01	0.07	0.01
$\tilde{U}_i$ (units)	(3000,5000,6000)	(12000,14000,17000)	(4000,6000,8000)	(1000,3000,5000)
r_i	0.88	0.91	0.97	0.85
f_i	0.02	0.01	0.06	0.04
B_i ($)	25000	100000	35000	5500
$\tilde{D} = (180000, 20000, 220000)$, $\tilde{F} = (540, 600, 660)$, $\tilde{P} = (16500, 18400, 20240)$.				

Table 2. Vendor source data for the problem at α=0.3.

Vendor No.	1	2	3	4
$(\tilde{s}_i)_{\alpha=0.3}$ ($)	[2.3, 5.1]	[1.3, 4.1]	[4.9, 8.4]	[0.3, 1.7]
q_i (%)	0.05	0.03	0	0.02
l_i (%)	0.03	0.01	0.07	0.01
$(\tilde{U}_i)_{\alpha=0.3}$(units)	[3600, 5700]	[12600,16100]	[4600,7400]	[1600,4400]
r_i	0.88	0.91	0.97	0.85
f_i	0.02	0.01	0.06	0.04
$\tilde{}B_i$ ($)	25000	100000	35000	5500
$(\tilde{D})_{\alpha=0.3} \in [18600, 21400]$, $(\tilde{P})_{\alpha=0.3} \in [17070, 19688]$, $(\tilde{F})_{\alpha=0,3} \in [558, 642]$.				

Step 1. $57000 \le Z_1 \le 71833.34,\ 413.33 \le Z_2 \le 521.66,\ 604.16 \le Z_3 \le 816.66.$

Step 2. Let $\alpha = 0.3$.

Step 3. Solve the following problem:

$$
\begin{aligned}
&\underset{x}{\text{Max}}\ \nu, \\
&\text{subject to} \\
&\quad 2.3x_1 + 1.3x_2 + 4.9x_3 + 0.3x_4 + 14833.34\nu \le 71833.34, \\
&\quad 0.05x_1 + 0.03x_2 + 0x_3 + 0.02x_4 + 108.33\nu \le 521.66, \\
&\quad 0.03x_1 + 0.01x_2 + 0.07x_3 + 0.01x_4 + 212.5\nu \le 816.66, \\
&\quad x_1 + x_2 + x_3 + x_4 \le 21400, \\
&\quad x_1 \le 5700, \\
&\quad x_2 \le 16100, \\
&\quad x_3 \le 7400, \\
&\quad x_4 \le 4400, \\
&\quad 0.88x_1 + 0.91x_2 + 0.97x_3 + 0.85x_4 \ge 17070, \\
&\quad 0.02x_1 + 0.01x_2 + 0.06x_3 + 0.04x_4 \le 640, \\
&\quad 2.3x_1 \le 25000, \\
&\quad 1.3x_2 \le 100000, \\
&\quad 4.9x_3 \le 35000, \\
&\quad 0.3x_4 \le 5500, \\
&\quad 0 \le \nu \le 1, \\
&\quad x_1, x_2, x_3, x_4 \ge 0, \text{ and integers.}
\end{aligned}
\tag{13}
$$

The $\alpha-$optimal compromise solution is $x_1 = 0$, $x_2 = 14092$, $x_3 = 4621$, $x_4 = 1287$, and $\nu = 0.678$.

8 | Concluding Remarks and Future Works

This paper investigates an interactive fuzzy programming approach for solving VSPs with fuzzy numbers in the price of a unit item, the upper limit of the quantity available, and aggregate demand for the items. After converting the fuzzy VSP into an equivalent deterministic VSP, a fuzzy programming approach has been applied by defining a linear membership function. An interactive procedure for obtaining α-optimal compromise solution has been presented. An illustrative numerical example has been given to clarify the obtained results. Future works might contain the additional extension of this study to other fuzzy-like structures (i.e., Neutrosophic set, interval-valued fuzzy set, Spherical fuzzy set, Pythagorean fuzzy set, etc. In addition, one can consider new fuzzy systems such as interval type-2, interval type-3, Possibility Interval-valued Intuitionistic fuzzy set, Possibility Neutrosophic set, Possibility Interval-valued Neutrosophic set, Possibility Interval-valued fuzzy set, Possibility fuzzy expert set, etc., with applications in decision-making.

Author Contributions

The authors equally contributed to the study.

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Data Availability

All the data are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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