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A HyF-WASPAS Method based on a Similarity Measure to Access Renewable Energy Technologies

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Abstract

Population growth and technological advancements have progressively expanded the demand for global energy. Growing public awareness about environmental problems, as well as the depletion of fossil resources, has prompted a worldwide shift to renewable energy sources. Renewable energy conveys an adaptable and sustainable approach for meeting future demands. The growing use of renewable energy reinforces a significant combination of economic benefits, social progress, environmental concerns, and technological advancements in sustainable solutions. Determining an efficient Renewable Energy Technology (RET) is a challenging task for decision-makers, as it involves a variety of sustainability factors that create uncertainty. The present research aims to offer a new, robust framework for evaluating RETs from a sustainable perspective by employing a novel similarity measure and the Weighted Aggregated Sum Product Assessment (WASPAS) approach in a Hyperbolic Fuzzy decision environment. Although several studies have recently contributed to the evaluation of RET, no research has investigated RET within the HyF framework, nor has the existing literature examined similarity measures on Hyperbolic Fuzzy Sets (HyFSs), which can effectively handle optimistic and pessimistic grades independently. Consequently, this paper firstly develops a novel similarity measure for HyFSs that efficaciously addresses all the axiomatic definitions and fundamental properties of a similarity measure on HyFSs. Moreover, its superiority and reliability are demonstrated through a comparative analysis with the existing similarity measures. Next, the WASPAS method is extended to solve Multiple-Criteria Decision Making (MCDM) problems in HyFSs. Furthermore, we apply this proposed methodology to select the most effective RET, comparing it to existing MCDM methods to validate its reliability and consistency. The experimental results indicate that the proposed MCDM methodology successfully determines RETs in a Hyperbolic Fuzzy environment and exhibits higher consistency compared to existing methods.

Keywords: Hyperbolic fuzzy sets, Renewable energy technologies, Similarity measure, Weighted aggregated sum product assessment, Multiple-criteria decision making.

1 | Introduction

Real-world situations frequently involve uncertainty arising from incomplete understanding, ambiguity, limited practical information, and data inaccuracies. To address such challenges, Zadeh [1] introduced Fuzzy

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Set Theory (FST), which supports decision-making that mirrors human reasoning. FST is well-suited for situations where binary logic cannot adequately represent the complexity involved. In this theory, a membership function is used to model uncertain information in decision-making problems. However, many real-life scenarios involve not only a degree of preference (membership) but also non-preference (non-membership) and hesitation. To capture this, Atanassov [2] proposed Intuitionistic Fuzzy Set Theory (IFST), which defines each element with a membership degree, a non-membership degree, and a hesitation degree, constrained such that their sum equals one. While IFST effectively addresses various uncertain conditions, it falls short in handling more complex situations. To overcome this limitation, Yager [3] introduced Pythagorean Fuzzy Set Theory (PFST), in which the sum of the squares of the membership, non-membership, and hesitation degrees must not exceed one, offering a more flexible approach to modelling uncertainty. Subsequently, Yager [4] introduced Q-Rung Orthopair Fuzzy Set Theory (Q-ROFST), and Senapati and Yager [5] developed Fermatean Fuzzy Set Theory (FFST) for even greater flexibility. FFST is a special case of Q-ROFST where $q = 3$, requiring that the sum of the qth powers of the membership and non-membership degrees be limited to one. Higher q values allow broader ranges for membership grades, making these models particularly effective for real-world applications such as decision-making and pattern recognition. Despite their advancements, models like Intuitionistic Fuzzy Set (IFS), PFS, FFS, and Q-ROFS sometimes cannot represent situations where it is intuitive to assign full membership and partial non-membership (or vice versa). In such cases, the ability to independently determine membership and non-membership values becomes crucial for accurate decision-making. For example, during the COVID-19 pandemic, treatments showed effective short-term results, but their long-term effects remained uncertain. Understanding these long-term consequences is essential, especially for individuals known as "long-haulers" who experience persistent symptoms such as fatigue and joint pain. Although vaccines helped control the pandemic, their lasting health effects were not fully known. This scenario requires an optimistic degree of one (for short-term effectiveness) and a pessimistic degree between zero and one (for uncertain long-term effects), which existing fuzzy models cannot handle effectively. To manage such paradoxical and complex real-life situations, Dutta and Borah [6] introduced the Hyperbolic Fuzzy Set (HyFS). This model uses two independent values—an optimistic degree and a pessimistic degree—each within the range [0,1], allowing for a more nuanced and accurate representation of uncertainty. HyFS offers several advantages over other Fuzzy Set (FS) extensions, including improved flexibility, expressiveness, reliability, and computational efficiency, making it highly suitable for uncertain decision-making tasks. *Fig. 1* illustrates the graphical representation of membership and non-membership functions for IFS, PFS, FFS, Q-ROFS, and HyFS, while *Table 1* provides a structural comparison of these models. The discussion then shifts to the significance of renewable energy, which is vital today for mitigating climate change, reducing dependence on finite fossil fuels, and ensuring long-term energy security. Renewable sources such as solar, wind, hydro, and geothermal produce minimal greenhouse gas emissions, helping to reduce environmental degradation. Unlike fossil fuels, renewables offer a clean, inexhaustible energy supply that can be locally sourced, thus enhancing national energy independence. Economically, the renewable energy sector generates millions of jobs and helps lower healthcare costs by reducing pollution-related illnesses. From a sustainability standpoint, renewables support environmental goals by protecting ecosystems and cutting emissions; economic goals by reducing long-term costs and promoting innovation; and social goals by improving public health, expanding energy access, and ensuring equity. Therefore, transitioning to renewable energy is essential for achieving a cleaner, healthier, and more sustainable future. Effective decision-making is crucial in selecting the most appropriate renewable energy source, taking into account social, economic, environmental, and resource-related factors. Since each renewable source has its own strengths and limitations, it is important to evaluate them based on geography, climate, cost, infrastructure, and long-term impact. A well-structured decision-making process—incorporating feasibility studies, cost-benefit analysis, and sustainability planning—ensures efficient and responsible adoption of renewable energy technologies. The main goal of this study is to develop a Multiple-Criteria Decision Making (MCDM) approach using the HyFS framework to determine the optimal Renewable Energy Technology (RET) for specific needs.

To meet the objectives, the remaining section of the study is structured as follows. Section 2 contains a review of the existing literature related to this study. Section 3 comprises the preliminaries on FSs and their extensions. Section 4 introduces a novel HyF similarity measure and discusses its properties in the HyF framework. Additionally, this section presents some notable existing similarity measures for IFS, PFS, and FFS.

Furthermore, we conduct a superiority check of our proposed similarity measure against existing ones related to this study. In Section 5, we present a novel HyF-VIKOR MCDM method, supported by the proposed HyF similarity measure introduced in the previous section. In Section 6, we utilize the novel HyF-WASPAS MCDM in a case study on RET selection and perform a comparative analysis to verify the reliability and consistency of our proposed methodology with existing MCDM methods. Finally, the conclusion and future studies of this article are presented in Section 7.

Table 1. Structural presentation of IFS, PFS, FFS, Q-ROFS, and HyFS.

IFS	PFS	FFS	Q-ROFS	HyFS
$0 \leq \alpha + \beta \leq 1$	$0 \leq \alpha^2 + \beta^2 \leq 1$	$0 \leq \alpha^3 + \beta^3 \leq 1$	$0 \leq \alpha^q + \beta^q \leq 1$	$0 \leq \alpha \leq 1; 0 \leq \beta \leq 1$
$\pi = 1 - (\alpha + \beta)$	$\pi = \sqrt{1 - (\alpha^2 + \beta^2)}$	$\pi = \sqrt[3]{1 - (\alpha^3 + \beta^3)}$	$\pi = \sqrt[q]{1 - (\alpha^q + \beta^q)}$	No hesitancy
$\alpha + \beta + \pi = 1$	$\alpha^2 + \beta^2 + \pi^2 = 1$	$\alpha^3 + \beta^3 + \pi^3 = 1$	$\alpha^q + \beta^q + \pi^q = 1$	$\alpha\beta = 1$

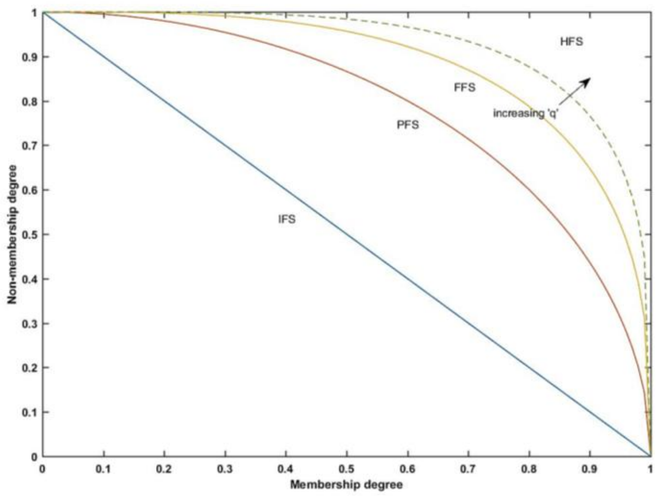

Fig. 1. Membership and non-membership grades of IFS, PFS, FFS, and HyFS.

2 | Literature Review

A literature review is a crucial component of research. This study presents a concise literature review on similarity measures, the Weighted Aggregated Sum Product Assessment (WASPAS) MCDM method, and the existing decision-making procedures on RET selection.

A similarity measure is a mathematical tool used to quantify the degree of similarity between two objects or data sets. Similarity metrics are crucial for informed decision-making, as they enable decisions based on data, categorize similar options, and reveal underlying patterns. Some of the notable similarity measures from the literature are Peng et al. [7], Wei and Wei [8], Song et al. [9], Jiang et al. [10], Gohain et al. [11], Wang et al. [12].

WASPAS MCDM approach is a ranking method that combines the Weighted Sum Model (WSM) and Weighted Product Model (WPM). Stanujkić et al. [13] conducted a website evaluation, and Mishra et al. [14] assessed cellular mobile telephone providers by the WASPAS method under an intuitionistic framework for MCDM problems. Kahraman et al. [15] selected GSM operators, and Ilbahar et al. [16] prioritized renewable energy sources by the application of the WASPAS approach under PFSs. Keshavarz-Ghorabaee et al. [17]

evaluated green construction suppliers, and Mishra & Rani [18] optimized health care waste disposal selection under the Fermatean fuzzy WASPAS approach.

Choosing the best RET requires careful consideration of a number of environmental, economic, social, and technological variables. In the literature, researchers have employed numerous MCDM approaches for optimal RET selection. Rani et al. [19] conducted the VIKOR approach to evaluate RET in India under PFSs. Krishankumar et al. [19] evaluated RET in India with partial weight information. Gupta et al. [21] determined the best renewable energy source under the VIKOR method in the IFS context. Sitorus and Brito-Parada [20] selected ideal renewable energy technologies using a hybrid subjective and objective MCDM method.

The key objectives of this research study are outlined below:

I. Introduction of a novel similarity measure on the HyF framework.

II. Perform a comparative analysis of the proposed HyF similarity measure with some notable existing similarity measures to test the superiority and reliability of the proposed similarity measure.

III. Introduction of the HyF-WASPAS approach.

IV. Application of an MCDM problem on the selection of renewable energy in the HyFS environment.

V. Computation of criteria weights based on the proposed similarity measure and evaluation of the optimal ranking of renewable energy by the HyF-WASPAS method.

VI. Comparative analysis of the extended HyF-WASPAS method in the HyF environment with existing MCDM methods to validate the robustness and precision of the method.

3 | Preliminaries

This section provides a brief overview of FS, IFS, PFS, Q-ROFS, FFS, and HyFS, as well as presents the basic operational principles of HyFS.

Definition 1 ([1]). On a finite universe of discourse X, an FS, F on X is specified as $F = \{\langle x, \alpha_F(x): x \in X\rangle\}$ such that, $\alpha_F(x): X \longrightarrow [0,1]$ represents the membership function of the element $x \in X$ to F.

Definition 2 ([2]). On a finite universe of discourse X, an Intuitionistic Fuzzy Set (IFS), $I = \{\langle x, \alpha_I(x), \beta_I(x): x \in X\rangle\}$ is defined by a membership function $\alpha_I: X \longrightarrow [0,1]$ and a non-membership function $\beta_I : X \longrightarrow [0,1]$ of the element $x \in X$ to the set I, such that $0 \le \alpha_I(x) + \beta_I(x) \le 1$ and $\pi_I(x) = 1 - \alpha_I(x) - \beta_I(x)$ signifies the hesitancy degree of the element $x \in X$ to the set I.

Definition 3 ([3]). On a finite universe of discourse X, a Pythagorean Fuzzy Set (PFS), $P = \{\langle x, \alpha_P(x), \beta_P(x): x \in X\rangle\}$ is defined by a membership function $\alpha_P: X \longrightarrow [0,1]$ and a non-membership function $\beta_P : X \longrightarrow [0,1]$ of the element $x \in X$ to the set I, such that $0 \le \alpha_P^2(x) + \beta_P^2(x) \le 1$ and $\pi_P(x) = 1 - \alpha_P^2(x) - \beta_P^2(x)$ signifies the hesitancy degree of the element $x \in X$ to the set P.

Definition 4 ([4]). On a finite universe of discourse X, a Q-rung Orthopair Fuzzy Set (Q-ROFS), $P = \{\langle x, \alpha_P(x), \beta_P(x): x \in X\rangle\}$ is defined by a membership function $\alpha_P: X \longrightarrow [0,1]$ and a non-membership function $\beta_P : X \longrightarrow [0,1]$ of the element $x \in X$ to the set I, such that $0 \le \alpha_P^2(x) + \beta_P^2(x) \le 1$ and $\pi_P(x) = 1 - \alpha_P^2(x) - \beta_P^2(x)$ signifies the hesitancy degree of the element $x \in X$ to the set P.

Definition 5 ([5]). On a finite universe of discourse X, a Fermatean Fuzzy Set (FFS), $P = \{\langle x, \alpha_P(x), \beta_P(x): x \in X\rangle\}$ is defined by a membership function $\alpha_P: X \longrightarrow [0,1]$ and a non-membership function $\beta_P : X \longrightarrow [0,1]$ of the element $x \in X$ to the set I, such that $0 \le \alpha_P^2(x) + \beta_P^2(x) \le 1$ and $\pi_P(x) = 1 - \alpha_P^2(x) - \beta_P^2(x)$ signifies the hesitancy degree of the element $x \in X$ to the set P.

Definition 6 (HyFS) ([6]). On a finite universe of discourse X, a HyFS, $H = \{(x, \alpha_H(x_i), \beta_H(x_i)): x_i \in X\}$, on X is characterized by an optimistic degree $\alpha_H(x_i): Z \longrightarrow [0,1]$ and a pessimistic degree $\beta_H(x_i) : X \longrightarrow [0,1]$ of $x_i \in X$ such that the optimistic and pessimistic degrees are independent of each other, with the property $0 \le (\alpha_H(x_i))(\beta_H(x_i))) \le 1$ for all $x_i \in X$. We signify a HyFS on a finite universe of discourse X as $HyFS(X)$

Definition 7. The operational laws on HyFS ([6]). For any three $H = \{(x, \alpha_H(x_i), \beta_H(x_i)) : x_i \in X\}, H_1 = \{(x, \alpha_{H_1}(x_i), \beta_{H_1}(x_i)) : x_i \in X\}, H_2 = \{(x, \alpha_{H_2}(x_i), \beta_{H_2}(x_i)) : x_i \in X\} \in HyFS(X)$, the fundamental set operations are defined as follows:

I. $H^c = \{ (x, 1 - \alpha_H(x_i), 1 - \beta_H(x_i)) : x \in X\}$.

II. $(H^c)^c = H^c$.

III. $H_1 \subseteq H_2$ if and only if $\alpha_{H_1}(x_i) \leq \alpha_{H_2}(x_i)$ and $\beta_{H_1}(x_i) \geq \beta_{H_2}(x_i)$.

IV. $H_1 = H_2$ if and only if $\alpha_{H_1}(x_i) \subseteq \alpha_{H_2}(x_i)$and $\beta_{H_1}(x_i) \supseteq \beta_{H_2}(x_i)$.

V. $H_1 \wedge H_2 = \left\{\left(x, \ \min\left(\alpha_{H_1}(x_i), \alpha_{H_2}(x_i)\right), \max\left(\beta_{H_1}(x_i), \beta_{H_2}(x_i)\right)\right) : x \in X\right\}$.

VI. $H_1 \vee H_2 = \left\{\left(x, \ \max\left(\alpha_{H_1}(x_i), \alpha_{H_2}(x_i)\right), \min\left(\beta_{H_1}(x_i), \beta_{H_2}(x_i)\right)\right) : x \in X\right\}$.

Definition 8 ([6]). For a $H \in HyFS(X)$, the score $S(H)$ and the accuracy function $A(H)$ is defined respectively as

$$S(H) = 2\alpha_H(x) - \alpha_H(x)\beta_H(x), \text{where } S(H) \in [0,2].$$
$$A(H) = 2\alpha_H(w) - \alpha_H(w)\beta_H(x), \text{where } A(H) \in [0,2].$$

4 | A Novel HyF Similarity Measure

Similarity measure plays a significant role in assessing the similarity of uncertain information in a diverse decision-making environment. This section highlights some notable similarity measures. Additionally, we introduce a novel similarity measure on HyFS and establish several properties and theorems associated with this measure. Furthermore, the proposed similarity measure is compared with existing measures to demonstrate its superiority and reliability.

4.1 | Existing Similarity Measures

In the literature, researchers worldwide have developed various similarity measures to quantify ambiguous information. In this subsection, we discuss some previously established similarity measures on IFS, PFS, FFS, and Q-ROFS from the literature.

Consider, $H_1 = \{(x, \alpha_{H_1}(x_i), \beta_{H_1}(x_i)) : x_i \in X\}$, $H_2 = \{(x, \alpha_{H_2}(x_i), \beta_{H_2}(x_i)) : x_i \in X\} \in HyFS(X)$, where $X = \{x_i : i = 1,2,3 \dots, n\}$ then clearly H_1, H_2 are also IFS, PFS, FFS, and Q-ROFS. *Table 2* displays the existing similarity measures between H_1 and H_2 as follows.

Table 2. Existing similarity measures.

Authors	Similarity Measure
Song et al. [9]	$S_{So}(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left[2\sqrt{\alpha_A(x_i)\alpha_B(x_i)} + 2\sqrt{\beta_A(x_i)\beta_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-\alpha_A(x_i))(1-\alpha_B(x_i))} + \sqrt{(1-\beta_A(x_i))(1-\beta_B(x_i))}\right]$.
Jiang et al. [10]	$S_J(A,B) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(\left\lvert\frac{2(\alpha_A(x_i)\pi_B(x_i) - \pi_A(x_i)\alpha_B(x_i)) - 4(\alpha_A(x_i) - \alpha_B(x_i))}{4 - \pi_A(x_i)\pi_B(x_i)}\right\rvert + \left\lvert\frac{2(\beta_A(x_i)\pi_B(x_i) - \pi_A(x_i)\beta_B(x_i)) + 4(\beta_A(x_i) - \beta_B(x_i)) + 2(\pi_A(x_i) - \pi_B(x_i))}{4 - \pi_A(x_i)\pi_B(x_i)}\right\rvert\right)$.

Table 2. Continued.

Authors	Similarity Measure
Gohain et al. [11]	$S_G(A,B) = 1 - \left[\frac{1}{12n}\sum_{i=1}^{n}\{(\lvert\alpha_A(x_i) - \alpha_B(x_i)\rvert^2 + \lvert\beta_A(x_i) - \beta_B(x_i)\rvert^2)(2 - \lvert\pi_A(x_i) - \pi_B(x_i)\rvert)^2 + 2(\lvert\min\{\alpha_A(x_i)\beta_B(x_i)\} - \min\{\alpha_A(x_i)\beta_B(x_i)\}\rvert^2 + \lvert\max\{\alpha_A(x_i)\beta_B(x_i)\} - \max\{\alpha_A(x_i)\beta_B(x_i)\}\rvert^2)\}\right]^{\frac{1}{2}}.$
Peng et al. [7]	$S_P(A,B) = 1 - \frac{\sum_{i=1}^{n}\left\lvert\left(\alpha_A^2(x_i) - \beta_A^2(x_i)\right) - \left(\alpha_B^2(x_i) - \beta_B^2(x_i)\right)\right\rvert}{2n}.$
Wei and Wei [8]	$S_{WW}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\alpha_A^2(x_i)\alpha_B^2(x_i) + \beta_A^2(x_i)\beta_B^2(x_i)}{\sqrt{\alpha_A^4(x_i) + \beta_A^4(x_i)} + \sqrt{\alpha_B^4(x_i) + \beta_B^4(x_i)}}.$
Wang et al. [12]	$S_{Wa}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\alpha_A^3(x_i)\alpha_B^3(x_i) + \beta_A^3(x_i)\beta_B^3(x_i)}{\alpha_A^6(x_i) + \beta_A^6(x_i) + \alpha_A^6(x_i) + \beta_A^6(x_i) - \alpha_A^3(x_i)\alpha_B^3(x_i) - \beta_A^3(x_i)\beta_B^3(x_i)}.$

4.2 | Proposed Novel Similarity Measure on HyFSs

In this subsection, a novel similarity measure for HyFSs is constructed based on the concept of differences in optimistic and pessimistic degrees between two HyFSs, along with the cross-evaluation of these degrees between them. The implementation of the cross-evaluation factor in the proposed Hyperbolic Fuzzy similarity measure empowers decision makers to precisely evaluate human judgments while gathering accurate information without information loss in diverse uncertain decision-making situations. Following this, the fundamental properties and propositions for the proposed similarity measure are also discussed in this subsection.

Definition 9. Given a finite universe of discourse $X = \{x_i : i = 1,2,3 \dots, n\}$, let $H_1 = \{(x_i, \alpha_{H_1}(x_i), \beta_{H_1}(x_i)) : x_i \in X\}$, $H_2 = \{(x_i, \alpha_{H_2}(x_i), \beta_{H_1}(x_i) : x_i \in X) : x_i \in X\}$ be two HyFSs, then we define the novel similarity measure between H_1 and H_2 as

$$S_{HyFKD}(H_1, H_2) = 1 - \frac{1}{3n}\sum_{i=1}^{n}\left[\frac{\lvert\alpha_{H_1}(x_i) - \alpha_{H_2}(x_i)\rvert}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right)} + \frac{\lvert\beta_{H_1}(x_i) - \beta_{H_2}(x_i)\rvert}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_2}(x_i)\right)} + \left\lvert\alpha_{H_1}(x_i)\beta_{H_2}(x_i) - \beta_{H_1}(x_i)\alpha_{H_2}(x_i)\right\rvert\right],$$

and 'n' is the number of pairs of HyFSs.

Definition 10. Let $H_1 = \{(x_i, \alpha_{H_1}(x_i), \beta_{H_1}(x_i)) : x_i \in X\}$, $H_2 = \{(x_i, \alpha_{H_2}(x_i), \beta_{H_1}(x_i) : x_i \in X) : x_i \in X\}$ and $H_3 = \{(x_i, \alpha_{H_3}(x_i), \beta_{H_3}(x_i)) : x_i \in X\}$ be three HyFSs on a finite universe of discourse $X = \{x_i : i = 1,2,3 \dots, n\}$.

Then the novel HyF similarity measure S_{HyFKD}, satisfies the following properties:

I. $P_1: 0 \leq S_{HyFKD}(H_1, H_2) \leq 1$.

II. $P_2: S_{HyFKD} = 0$ if and only if $H_1 = H_2$.

III. $P_3: S_{HyFKD}(H_1, H_2) = S_{HyFKD}(H_2, H_1)$.

IV. P_4: For $H_1 \subseteq H_2 \subseteq H_3$, then $S_{HyFKD}(H_1, H_2) \geq S_{HyFKD}(H_1, H_3)$ and $S_{HyFKD}(H_2, H_3) \geq S_{HyFKD}(H_1, H_3)$.

Proof (P_1): Since, H_1, H_2 and H_3 are HyFSs, then clearly $0 \leq S_{HyPA}(H_1, H_2) \leq 1$.

Proof (P_2): Let, $S_{HyPA}(H_1, H_2) = 0$, then for all $i = 1,2, \dots, n$.

We have

$$\frac{1}{3}\left[\frac{|\alpha_{H_1}(x_i)-\alpha_{H_2}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right)}+\frac{|\beta_{H_1}(x_i)-\beta_{H_2}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_2}(x_i)\right)}+\left|\alpha_{H_1}(x_i)\beta_{H_2}(x_i)-\beta_{H_1}(x_i)\alpha_{H_2}(x_i)\right|\right]=0 \quad (1)$$

$$\Leftrightarrow \alpha_{H_1}(x_i)=\alpha_{H_2}(x_i) \text{ and } \beta_{H_1}(x_i)-\beta_{H_2}(x_i) \Leftrightarrow H_1 = H_2.$$

Proof (P_3): Clearly, it is obvious that $S_{HyFKD}(H_1,H_2) = S_{HyFKD}(H_2,H_1)$.

Proof (P_4): Consider, $H_1 \subseteq H_2 \subseteq H_3$.

Then, for all $i = 1,2,\dots,n$ we have, $0 \le \alpha_{H_1}(x_i) \le \alpha_{H_2}(x_i) \le \alpha_{H_3}(x_i) \le 1$ and $0 \ge \beta_{H_1}(x_i) \ge \beta_{H_2}(x_i) \ge \beta_{H_3}(x_i) \ge 1$.

It follows

$$\left|\alpha_{H_1}(x_i)-\alpha_{H_2}(x_i)\right| \le \left|\alpha_{H_1}(x_i)-\alpha_{H_3}(x_i)\right| \quad \text{and} \quad \left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right) \le \left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_3}(x_i)\right) \Rightarrow \left(\frac{|\alpha_{H_1}(x_i)-\alpha_{H_2}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right)}\right) \le \left(\frac{|\alpha_{H_1}(x_i)-\alpha_{H_3}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_3}(x_i)\right)}\right). \quad (1)$$

Similarly, we have

$$\left(\frac{|\beta_{H_1}(x_i)-\beta_{H_2}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_2}(x_i)\right)}\right) \ge \left(\frac{|\beta_{H_1}(x_i)-\beta_{H_3}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_3}(x_i)\right)}\right). \quad (2)$$

Also,

$$\alpha_{H_1}(x_i)\beta_{H_3}(x_i) \le \alpha_{H_1}(x_i)\beta_{H_2}(x_i) \le \alpha_{H_2}(x_i)\beta_1(x_i) \le \alpha_{H_3}(x_i)\beta_{H_1}(x_i)$$

$$\Rightarrow \left|\left(\alpha_{H_2}(x_i)\beta_1(x_i)\right)-\left(\alpha_{H_1}(x_i)\beta_{H_2}(x_i)\right)\right| \le \left|\left(\alpha_{H_3}(x_i)\beta_{H_1}(x_i)\right)-\left(\alpha_{H_1}(x_i)\beta_{H_3}(x_i)\right)\right|. \quad (3)$$

Adding (1), (2), (3), and multiplying by $\frac{1}{3}$ on both sides of the inequality for all $i = 1,2,\dots,n$, we obtain,

$$\frac{1}{3}\left[\frac{|\alpha_{H_1}(x_i)-\alpha_{H_2}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right)}+\frac{|\beta_{H_1}(x_i)-\beta_{H_2}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_2}(x_i)\right)}+\left|\alpha_{H_1}(x_i)\beta_{H_2}(x_i)-\beta_{H_1}(x_i)\alpha_{H_2}(x_i)\right|\right] \le \frac{1}{3}\left[\frac{|\alpha_{H_1}(x_i)-\alpha_{H_3}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_3}(x_i)\right)}+\frac{|\beta_{H_1}(x_i)-\beta_{H_3}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_3}(x_i)\right)}+\left|\alpha_{H_1}(x_i)\beta_{H_3}(x_i)-\beta_{H_1}(x_i)\alpha_{H_3}(x_i)\right|\right]. \quad (4)$$

Now taking summation for all $i = 1,2,3\dots n$, prefacing the factor $\frac{1}{n}$ on both sides of the *Inequality (4)* and then subtracting from 1 on both sides gives

$$1-\sum_{i=1}^{n}\left(\frac{1}{3}\left[\frac{|\alpha_{H_1}(x_i)-\alpha_{H_2}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_2}(x_i)\right)}+\frac{|\beta_{H_1}(x_i)-\beta_{H_2}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_2}(x_i)\right)}+\left|\alpha_{H_1}(x_i)\beta_{H_2}(x_i)-\beta_{H_1}(x_i)\alpha_{H_2}(x_i)\right|\right]\right) \ge 1-\sum_{i=}^{n}\left(\frac{1}{3}\left[\frac{|\alpha_{H_1}(x_i)-\alpha_{H_3}(x_i)|}{\left(1+\alpha_{H_1}(x_i)\right)\left(1+\alpha_{H_3}(x_i)\right)}+\frac{|\beta_{H_1}(x_i)-\beta_{H_3}(x_i)|}{\left(1+\beta_{H_1}(x_i)\right)\left(1+\beta_{H_3}(x_i)\right)}+\left|\alpha_{H_1}(x_i)\beta_{H_3}(x_i)-\beta_{H_1}(x_i)\alpha_{H_3}(x_i)\right|\right]\right).$$

Thus, we obtain $S_{HyFKD}(H_1,H_2) \ge S_{HyFKD}(H_1,H_3)$.

Similarly, we have $S_{HyFKD}(H_2,H_3) \ge S_{HyFKD}(H_1,H_3)$.

Proposition 1. For $H =< \alpha_H, \beta_H >$, $H^c =< 1-\alpha_H, 1-\beta_H >$, then

$$S_{HyFKD}(H,H^c) = 1-\frac{1}{3}\left[\frac{|2\alpha_H-1|}{(1+\alpha_H)(2-\alpha_H)}+\frac{|2\beta_H-1|}{(1+\beta_H)(2-\beta_H)}+|\alpha_H-\beta_H|\right].$$

Graphically, the similarity of $S_{HyFKD}(H, H^c)$ is shown in *Fig. 2*.

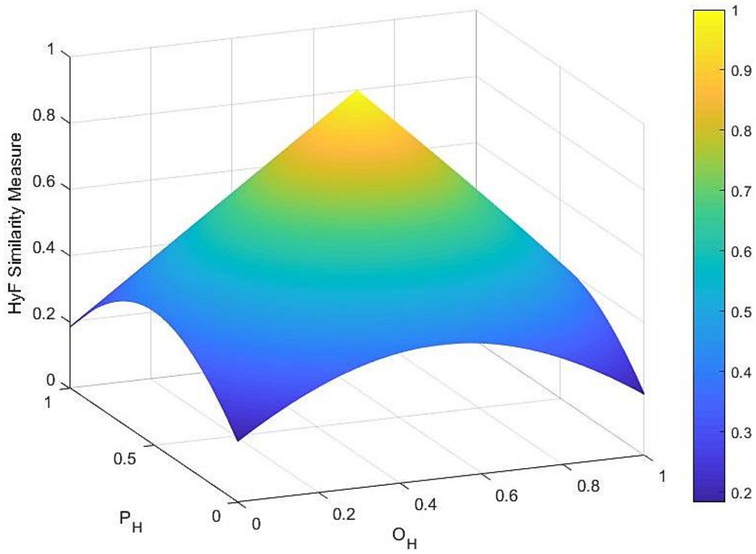

Fig. 2. Graphical representation of the similarity of $S_{HyFKD}(H, H^c)$.

Proposition 2. For $H =< \alpha_H, \alpha_H >$, $H_* =< \beta_H, 1-\beta_H >$, $H_*^c =< 1-\beta_H, \beta_H >$, then,

$$S_{HyFKD}(H, H_*) = 1 - \frac{1}{3}\left[\frac{|\alpha_H - \beta_H|}{(1+\alpha_H)(1+\beta_H)} + \frac{|\alpha_H - 1 + \beta_H|}{(1+\alpha_H)(2-\beta_H)} + |2\alpha_H\beta_H - \alpha_H|\right] = S_{HyFKD}(H, H_*^c).$$

Graphically, the similarity of $S_{HyFKD}(H, H_*) = S_{HyFKD}(H, H_*^c)$ is shown in *Fig. 3*.

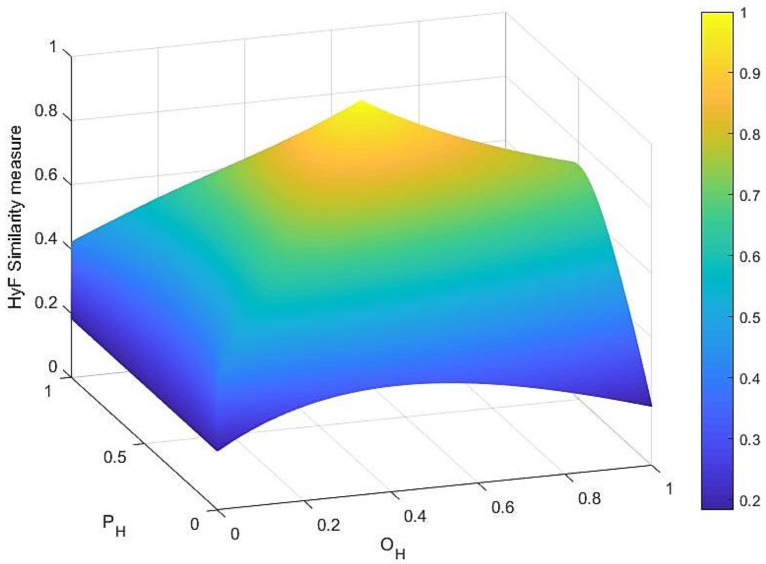

Fig. 3. Graphical representation of the similarity of $S_{HyFKD}(H, H_*) = S_{HyFKD}(H, H_*^c)$.

Proposition 3. For $H =< \alpha_H, 1-\alpha_H >$, $H_\# =< \beta_H, 1-\beta_H >$.

Then,

$$S_{HyFKD}(H, H_\#) = \frac{1}{3}\left[\frac{|\alpha_H - \beta_H|}{(1+\alpha_H)(1+\beta_H)} + \frac{|\alpha_H - \beta_H|}{(2-\alpha_H)(2-\beta_H)} + |\alpha_H - \beta_H|\right].$$

Graphically, the similarity of $S_{HyFKD}(H, H_\#)$ is shown in *Fig. 4*.

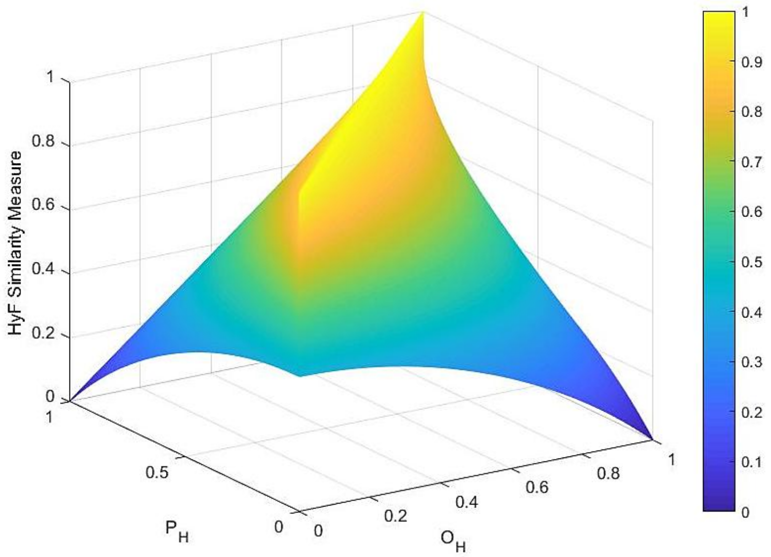

Fig. 4. Graphical representation of the similarity of $S_{HyFKD}(H, H_{\#})$.

Proposition 4. Consider the following three pairs of HyFSs:

I. $A = (0.7,0.3)$, $X = (x, 1 - x)$.

II. $B = (0.5,0.5)$, $X = (x, 1 - x)$.

III. $C = (0.3,0.7)$, $X = (x, 1 - x)$.

Then, the non-linear characteristics of the above three pairs of HyFSs are depicted in *Figs. 5-7*.

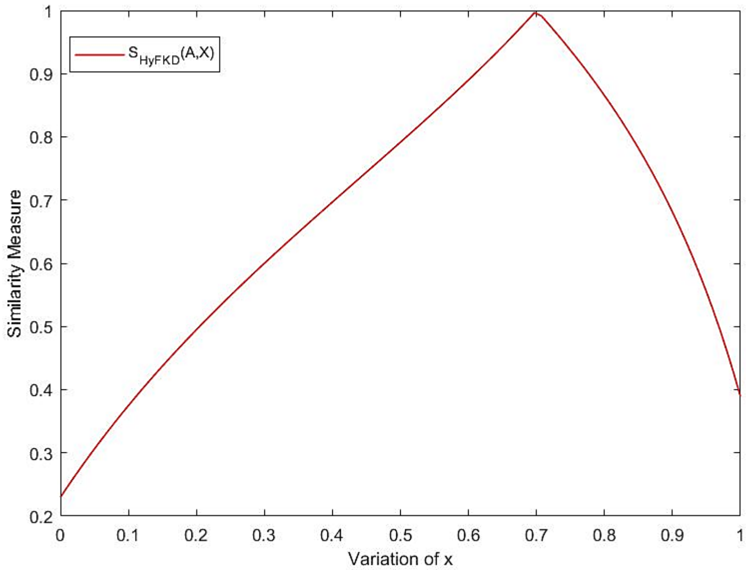

Fig. 5. Proposed similarity measure between $A = (0.7, 0.3)$ and $X = (x, 1 - x)$.

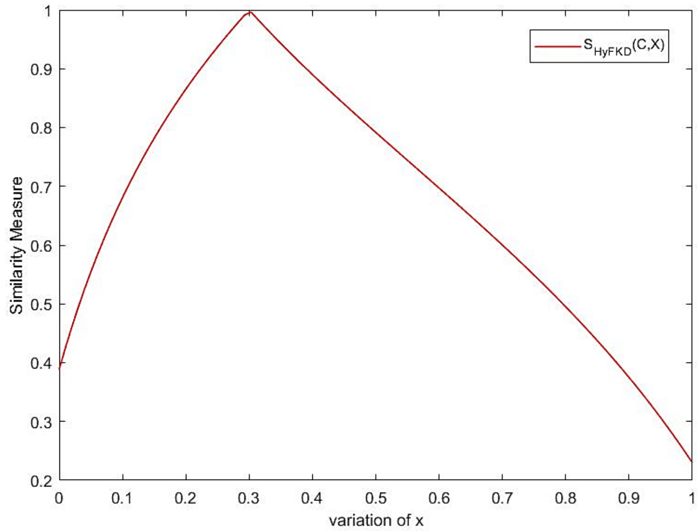

Fig. 6. Proposed similarity measure between $\mathbf{B = (0.5, 0.5)}$ and $\mathbf{X = (x, 1 - x)}$.

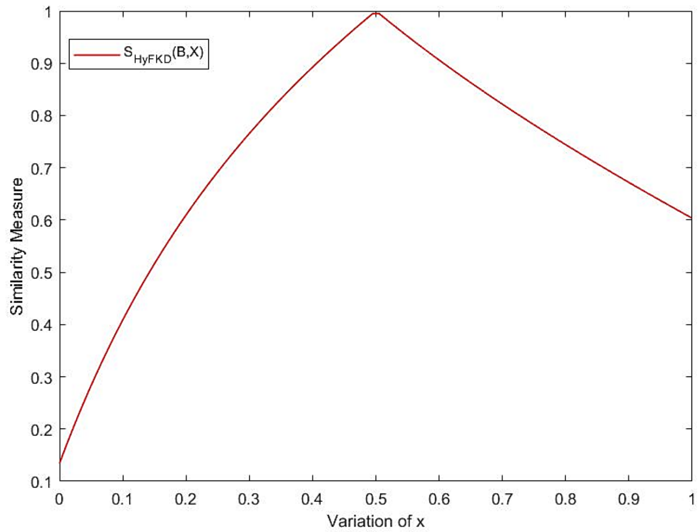

Fig. 7. Proposed similarity measure between $\mathbf{C = (0.3, 0.7)}$ and $\mathbf{X = (x, 1 - x)}$.

Comparison of similarity measures is an integral part of research work. In this section, we compare the proposed similarity measure with some notable existing similarity measures from the literature to show its superiority and reliability. In this regard, we adopted some pairs of FSs from Saikia et al. [21] to verify the superiority of the present approach, as shown in the following *Table 3*. Here, in *Table 3*, the bold data indicate contradictory outcomes. In contrast, our proposed novel similarity measure yields reliable and consistent outcomes.

Table 3. Comparison of similarity degrees under different fuzzy profiles.

$\mathbf{S_{HyFKD}(H_1, H_2)}$	**Profile 1**	**Profile 2**	**Profile 3**	**Profile 4**	**Profile 5**	**Profile 6**	**Profile 7**
Song et al. [9]	0.9882	0.9939	0.3333	0.4714	0.9877	0.9961	0.4690
Jiang et al. [10]	0.8979	0.9130	0.5	0.5	0.8979	0.9484	**0.5**
Gohain et al. [11]	0.9265	0.9	0.5	0.7958	0.9065	0.9316	0.7763
Peng et al. [7]	1	0.93	0.5	1	0.98	0.955	0.9
Wei and Wei [8]	1	0.8546	N/A	N/A	0.9949	0.9963	N/A
Wang et al. [12]	0.5579	0.5579	0	0	0.6680	0.6842	0
S_{HyFKD} (Proposed)	0.8787	0.7161	0.3333	0.5555	0.6765	0.7595	0.5

5 | Hyperbolic Fuzzy WASPAS Method based on Similarity Measure

In this section, we extend the traditional WASPAS method to the HyF environment, based on the proposed HyF framework. WASPAS MCDM approach is a ranking method that combines WSM and WPM. The extended WASPAS approach based on the proposed similarity measure under the HyF framework is presented as follows:

Step 1 (problem description). Suppose a set of decision makers $\{e_1, e_2, \dots e_k\}$ assesses a set of alternatives $\{A_1, A, \dots A_m\}$ based on the criteria set $\{C_1, C_2, \dots C_n\}$. Let every decision maker $e_r, r = 1,2, \dots, k$ constructs a decision matrix $Y^r = [y^r_{pq}]$, $r = 1,2, \dots, k, p = 1,2, \dots, m, q = 1,2, \dots, n$ for each alternative $A_p, p = 1,2, \dots, m$ in relation to a criterion C_q under the HyF environment, such that d^i_{pq} represents the judgment of the ith decision maker on the alternative a_p with reference to the criteria C_q.

Step 2. Compute the aggregated HyF decision matrix.

To determine the aggregated HyF decision matrix, we define a HyF Weighted Aggregation Operator (HyFWAO). Utilizing this HyFWAO, every individual decision matrix is combined into a group decision matrix to obtain the aggregated HyF decision matrix, $A = (y_{pq})_{m\times n}$, where

$$y_{pq} = (\alpha_{pq}, \beta_{pq}) = \text{HyFWGO}\left(y^{(1)}_{pq}, y^{(2)}_{pq}, \dots, y^{(k)}_{pq}\right) = \left(\prod_{r=1}^{k} u_r \alpha_{pq}, \prod_{r=1}^{k} u_r \beta_{pq}\right), p = 1,2, \dots, m, q = 1,2, \dots, n,$$

and $u_r > 0, r = 1,2, \dots, k$ represents the weight of each individual decision matrix such that $\sum_{r=1}^{k} u_r = 1$ is satisfied.

Step 3. Normalize the aggregated HyF decision matrix.

$$\zeta_{pq} = \begin{cases} (\alpha_{pq}, \beta_{pq}), & \text{q is a benefit criterion} \\ (\beta_{pq}, \alpha_{pq}), & \text{q is a cost criterion} \end{cases} = (\alpha^*_{pq}, \beta^*_{pq}).$$

Step 4. Compute criteria weights.

Calculate the weights of every criterion using the proposed similarity measure.

$$w_q = \frac{\sum_{p=1}^{m} \sum_{r=1, r\neq p}^{m} \left(1 - S_{HyFKD}(\zeta_{pq}, \zeta_{rq})\right)}{\sum_{q=1}^{n} \sum_{p=1}^{m} \sum_{r=1, r\neq p}^{m} \left(1 - S_{HyFKD}(\zeta_{pq}, \zeta_{rq})\right)}, q = 1(1)n.$$

Step 5. Compute the measures of WSM for each alternative using the formula:

$$\mathcal{Q}^{(1)}_p = \bigoplus_{q=1 \text{ to } n} (w_q \zeta_{pq}),$$

where, $\oplus_{q=1 \text{ to } n}(w_q y^N_{pq}) = \left[1 - \prod_{q=1}^{n}(1 - \alpha^*_{pq})^{w_q}, \prod_{q=1}^{n}(\beta^*_{pq})^{w_q}\right]$.

Step 6. Compute the measures of WPM for each alternative using the formula.

$$\mathcal{Q}^{(2)}_p = \bigotimes q = 1 \text{ to } n \, (w_q \zeta_{pq}),$$

where, $\otimes\, q = 1 \text{ to } n\, (w_q y^N_{pq}) = \left[\prod_{q=1}^{n}(\alpha^*_{pq})^{w_q}, 1 - \prod_{q=1}^{n}(1 - \beta^*_{pq})^{w_q}\right]$.

Step 7. Determine the aggregated measure or total importance, i.e., WASPAS measure of each alternative by computing.

$\mathcal{Q}_p = v\, \mathcal{Q}^{(1)}_p + (1 - v)\, \mathcal{Q}^{(2)}_p$, where v is the aggregating coefficient of decision precision such that $v \in [0,1]$.

Step 8. Rank the alternatives.

According to decreasing the crisp score values of $S(\mathcal{Q}_p)$, the alternatives are ranked.

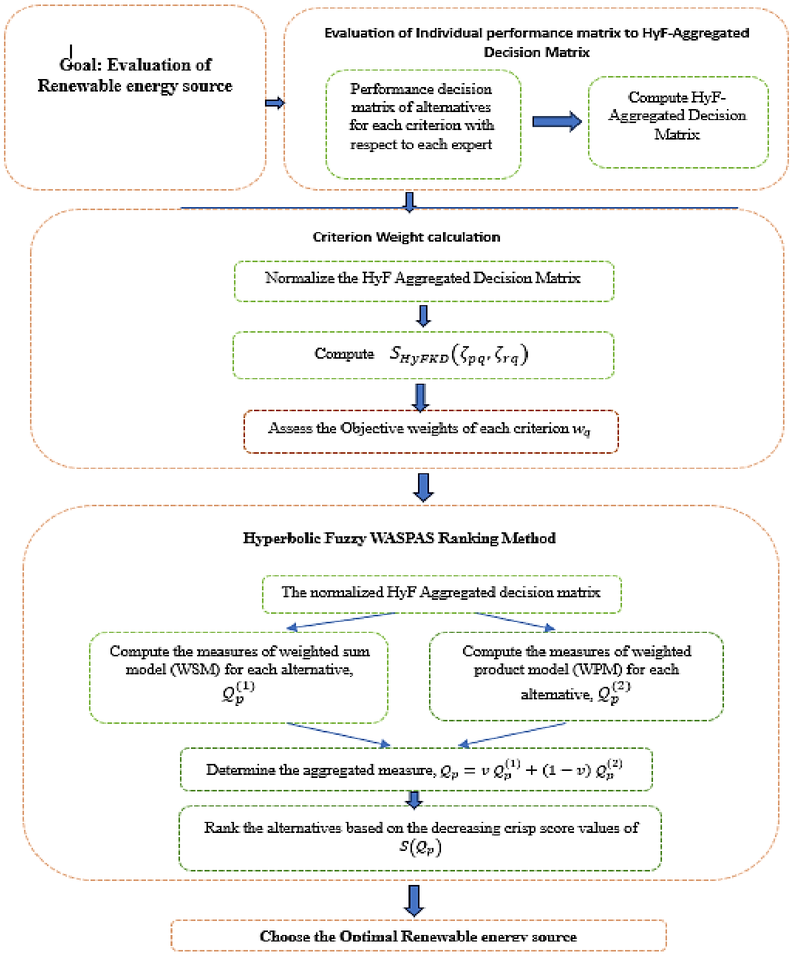

Fig. 8 Graphical flow chart of the HyF-WASPAS approach based on similarity measure.

6 | Application in Selection of Renewable Energy Technologies

The use of renewable energy over fossil fuels reduces climate change, conserves biodiversity, maximizes economic and technical benefits, and promotes social and health growth and equity. Selection of the optimal renewable energy source is a complex task and involves high uncertainty, as the process encompasses numerous social, economic, technical, and environmental criteria. We adopt a case study on the renewable energy selection problem from Krishankumar et al. [19]. The authors considered five renewable energy sources based on the criteria set by C with respect to the individual judgments of three decision makers.

$$C = \left\{ \begin{matrix} C_1: \text{Energy efficiency},\ C_2: \text{Job creation},\ C_3: \text{Complexity of technology},\ C_4: \text{Land usage}, \\ C_5: Co_2 \text{ emission},\ C_6: \text{Total cost}) \end{matrix} \right\}.$$

The criteria C_1, C_2 and C_4 are considered as benefit criteria and C_3, C_5 and C_6 are regarded as cost criteria. The individual performance matrix for evaluating renewable energy technologies is presented in *Table 4*. Next, utilizing the HyFWAO from *Step 2* of the previous algorithm and equal weights for each decision maker, such that $w_1 = 0.3333 = w_2 = w_3$, we obtain the HyF aggregated decision matrix for RET evaluation in *Table 5*. Utilizing *Step 4* based on the proposed similarity measure, we compute the criteria weights as

$$w_1 = 0.1946,\ w_2 = 0.1837,\ w_3 = 1671,\ w_4 = 0.1370,\ w_5 = 0.1748,\ w_5 = 0.1427.$$

Next, applying *Steps 5-8*, we obtain the WSM, $\mathcal{Q}_p^{(1)}$ the WPM, $\mathcal{Q}_p^{(2)}$ and the aggregated measure $\mathcal{Q}_p$ with $v = 0.5$ as shown in *Table 7*. Finally, rank the renewable energy sources based on the decreasing score values of $\mathcal{Q}_p$. Clearly, *Table 7* depicts that A_1 is the optimal renewable energy source.

Table 4. Individual decision matrices for renewable energy evaluation.

Decision Maker	Energy Sources	C_1	C_2	C_3	C_4	C_5	C_6
	A_1	(0.69,0.43)	(0.26,0.70)	(0.61,0.83)	(0.11,0.82)	(0.55.0.36)	(0.58,0.75)
	A_2	(0.28,0.31)	(0.53,0.59)	(0.66,0.39)	(0.14,0.24)	(0.26,0.57)	(0.51,0.47)
	A_3	(0.76,0.72)	(0.73,0.43)	(0.84,0.12)	(0.13,0.50)	(0.6,0.16)	(0.29,0.34)
	A_4	(0.13,0.43)	(0.46,0.69)	(0.66,0.62)	(0.41,0.88)	(0.46,0.45)	(0.74,0.24)
	A_5	(0.66,0.81)	(0.39,0.15)	(0.16,0.44)	(0.25,0.60)	(0.79,0.29)	(0.74,0.49)
	A_1	(0.63,0.16)	(0.20,0.88)	(0.87,0.29)	(0.54,0.54)	(0.42,0.77)	(0.37,0.14)
	A_2	(0.23,0.13)	(0.63,0.36)	(0.64,0.27)	(0.71,0.28)	(0.37.0.75)	(0.50,0.44)
	A_3	(0.84,0.28)	(0.12,0.85)	(0.35,0.48)	(0.34,0.72)	(0.46,0.60)	(0.52,0.69)
	A_4	(0.57,0.14)	(0.65,0.77)	(0.79,0.20)	(0.27,0.86)	(0.31,0.17)	(0.26,0.23)
	A_5	(0.86,0.50)	(0.50,0.65)	(0.69,0.67)	(0.50,0.36)	(0.11,0.012)	(0.45,0.17)
	A_1	(0.17,0.77)	(0.72,0.29)	(0.32,0.28)	(0.17,0.19)	(0.29,0.75)	(0.80,0.68)
	A_2	(0.48,0.67)	(0.22,0.30)	(0.17,0.52)	(0.86,0.67)	(0.86,0.39)	(0.27,0.66)
	A_3	(0.58,0.50)	(0.72,0.48)	(0.77,0.29)	(0.52,0.66)	(0.48,0.34)	(0.32,0.53)
	A_4	(0.83,0.41)	(0.58,0.57)	(0.76,0.48)	(0.44,0.13)	(0.84,0.36)	(0.34,0.34)
	A_5	(0.56,0.37)	(0.84,0.35)	(0.62,0.53)	(0.37,0.11)	(0.37,0.57)	(0.45,0.44)

Table 5. Aggregated decision matrix for renewable energy evaluation.

Energy Sources	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.696,0.293)	(0.486,0.433)	(0.403,0.676)	(0.336,0.573)	(0.549,0.353)	(0.659,0.399)
A_2	(0.559,0.236)	(0.693,0.296)	(0.419,0.529)	(0.213,0.513)	(0.453,0.423)	(0.569,0.356)
A_3	(0.619,0.473)	(0.676,0.309)	(0.479,0.439)	(0.343,0.466)	(0.633,0.219)	(0.463,0.346)
A_4	(0.509,0.276)	(0.586,0.396)	(0.453,0.539)	(0.336,0.693)	(0.553,0.349)	(0.646,0.279)
A_5	(0.586,0.503)	(0.563,0.213)	(0.286,0.513)	(0.316,0.566)	(0.632,0.213)	(0.679,0.329)

Table 6. Normalized aggregated decision matrix for renewable energy evaluation.

Energy Sources	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.696,0.293)	(0.486,0.433)	(0.676,0.403)	(0.336,0.573)	(0.353,0.549)	(0.399,0.659)
A_2	(0.559,0.236)	(0.693,0.296)	(0.529,0.419)	(0.213,0.513)	(0.423,0.453)	(0.356,0.569)
A_3	(0.619,0.473)	(0.676,0.309)	(0.439,0.479)	(0.343,0.466)	(0.219,0.633)	(0.346,0.463)
A_4	(0.509,0.276)	(0.586,0.396)	(0.539,0.453)	(0.336,0.693)	(0.349,0.553)	(0.279,0.646)
A_5	(0.586,0.503)	(0.563,0.213)	(0.513,0.286)	(0.316,0.566)	(0.213,0.632)	(0.329,0.679)

Table 7. Rank determination of renewable energy technologies.

Energy Sources	$\mathcal{Q}_p^{(1)}$	$\mathcal{Q}_p^{(2)}$	$\mathcal{Q}_p$	$S(\mathcal{Q}_p)$	Rank
A_1	(0.526,0.456)	(0.482,0.487)	(0.504,0.4172)	0.771	1
A_2	(0.500,0.383)	(0.451,0.413)	(0.476,0.398)	0.762	2
A_3	(0.480,0.459)	(0.421,0.479)	(0.450,0.469)	0.638	3
A_4	(0.456,0.463)	(0.428,0.507)	(0.443,0.485)	0.670	4
A_5	(0.448,0.4331)	(0.403,0.498)	(0.426,0.465)	0.654	5

Furthermore, to assess the robustness and consistency of our proposed MCDM approach, we conduct a comparative analysis with some notable existing MCM approaches, such as FF-TOPSIS [5], FF-ARAS [22], and FF-SAW [22]. *Table 8* presents the ranking of the renewable energy alternatives with respect to our proposed HyF-WASPAS approach and the other three existing methods. Clearly, *Table 8* displays that A_1 as the best RET under all the methods. *Fig. 9* gives a graphical representation of the ranking of the renewable energy alternatives.

Table 8. Comparison of the proposed HyF approach with existing fuzzy MCDM approaches.

MCDM Methods	Ranking Order
FF-TOPSIS [5]	$A_1 \geq A_2 \geq A_3 \geq A_5 \geq A_4$
FF-ARAS [22]	$A_1 \geq A_2 \geq A_3 \geq A_4 \geq A_5$
FF-SAW [22]	$A_1 \geq A_2 \geq A_3 \geq A_4 \geq A_5$
HyF-WASPAS (proposed)	$A_1 \geq A_2 \geq A_3 \geq A_4 \geq A_5$

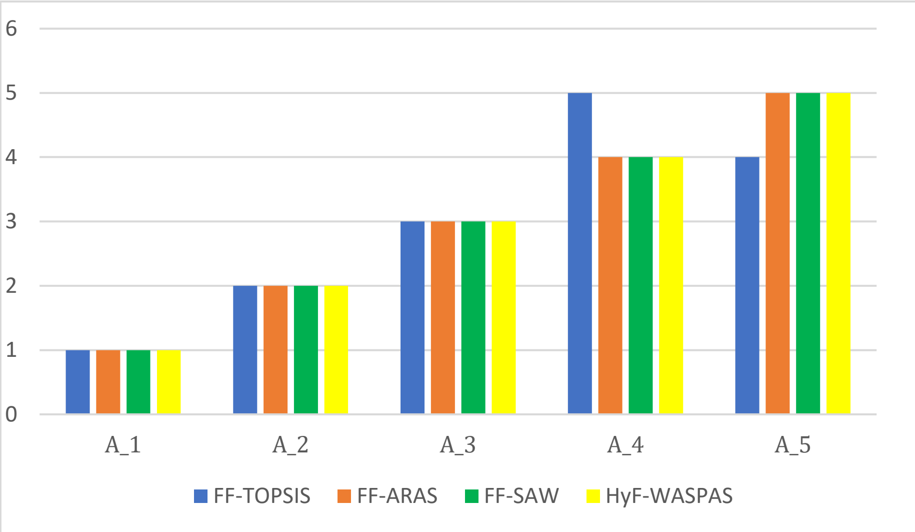

Fig. 9. Graphical comparison of the ranking of renewable energy alternatives.

7 | Conclusion

The mounting impacts of resource depletion, pollution, urbanisation, and climate change have significantly accelerated the requirement for renewable energy technologies. It focuses primarily on improving economic benefits, conserving natural resources, minimising environmental impact, and encouraging an equitable society while improving effectiveness and ensuring fair access to RET. The HyFS in this study can successfully overcome the limitations of FSs and their various extensions, such as IFS, PFS, Q-ROFS, and FFS. Correspondingly, the proposed HyF similarity measure successfully overcomes the limitations of some notable existing similarity measures from the literature. Moreover, decision-making in the HyF framework is more flexible compared to IFS, PFS, FFS, QFS, and QuFS, thus representing uncertainty and reducing information loss in a wide range of complex real-life applications. The extended WASPAS MCDM approach under the HyF framework in this study provides a reliable and consistent ranking of renewable energy alternatives in MCDM scenarios. The comparative analysis of our proposed HyF-WASPAS method, based on the novel similarity measure, with notable existing methods such as FF-TOPSIS [5], FF-ARAS [22], and FF-SAW [22], authenticates the reliability and consistency of our proposed approach. In the future, we will extend traditional MCDM methods, such as VIKOR ([25]), AHP ([26]), SAW ([27]), and ARAS ([28]), with our similarity measure under the HyF framework for handling complex MCDM problems.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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