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Measurement and Control of Insolvency and Inefficiency Risks

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Abstract

Unfortunately, there is currently no mathematician model who give some objective solution of the long-standing problem concerning the measurement and control of insolvency and inefficiency risks that runs a service producing company during a certain period of time. In this paper, we present a mathematical model, based on the fuzzy sets, which can be applied as a tool capable control and measure the risks of insolvency and inefficiency of a service producing company in a given period of time.
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1|Introduction

It well known that the applications of the fuzzy logic allows many kinds of designer and operator qualitative knowledge in system automation to be taken into account. Fuzzy logic began to interest the media at the beginning of the nineties. The numerous applications in electrical and electronic household appliances, particularly in Japan, were mainly responsible for such interest. Washing machines not requiring adjustment, camcorders with Steady shot (TM) image stabilization and many other innovations brought the term “fuzzy" logic to the attention of a wide public. In the car industry, automatic gear changes, injection and anti-rattle controls and air conditioning can be optimized thanks to fuzzy logic.

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In continuous and batch production processes, as well as in automation systems (which is the subject of this Cahier Technique), applications have also increased. Fuzzy logic has developed in this area as it is an essentially pragmatic, effective and generic approach. It allows systematization of empirical knowledge and which is thus hard to control. The theory of fuzzy sets offers a suitable method that is easy to implement in real time applications, and enables knowledge of designers and operators to be transcribed into dynamic control systems. This makes fuzzy logic able to tackle automation of procedures such as start-up and setting of parameters, for which few approaches were previously available. This Cahier Technique describes fuzzy logic and its application to production processes.

In the present day the fuzzy logic has found applications in many fields of different sciences such as:the physics, engineering, economics, social, and political sciences. In the particular case of the economic sciences, fuzzy logic has found applications in the disciplines of finance and business.

We know that the insolvency risk is the real possibility that a company may be unable to meet its payment obligations in a defined period of time – generally within a one-year horizon. It is also known as bankruptcy risk. The prediction of the insolvency risk risk have been the subject of much academic and professional research over the last half century. As when a pebble is thrown into a lake and the shock waves reach far beyond the point of initial impact, when a company becomes financially distressed/insolvent there are adverse consequences for its diverse stakeholder groups, such as investors, managers, employees, customers and suppliers, which impact onward into other firms, the wider economy and society.

In particular, those companies where the risk of insolvency during a certain period of time are immersed in deep uncertainty are of great interest, in other words, service producing companies. We are referring to those companies where both sales and costs at the beginning of the time period are only predictions. It is clear that a greater measurement of the risk of insolvency in that period of time implies an increase in efficiency in that lapse of time. Business efficiency describes how effectively a company generates products and services related to the amount of time and money needed to produce them. Efficient companies make the most of their resources, transforming labour, materials and capital into products and services that create profit for the company.

The term efficiency is very used in the context of finances of a company. In this work, we consider the efficiency of a company as a complex **system**. We assume that the main factor that makes the efficiency system of a company as complex results from the deep uncertainty that this system is submerged. All this made us think about the idea that every control model of the efficiency of a company seen as a system must be, in some way, a creative application of the fuzzy logic. In this paper, we show a control model, based on the fuzzy arithmetic, of the efficiency of a company seen as a system in period of time.

In this paper, we present a mathematical model, based on the fuzzy sets, which can be applied as a tool capable control and measure the risks of insolvency and inefficiency of a service producing company in a given period of time. The mathematical model that we present in this paper can be seen divided into three stages:

First Step: In this first step we intend to define the probabilities of inefficiency and insolvency, respectively, for the time period T.

Second Step: This second step concerns control and measurement systematic risk of insolvency and inefficiency in the period T. To achieve the main goal of this second part an extremely important measuring instrument is introduced to which we call rhythm differential.

Third Step: In this third step is classified the level of efficiency achieved by the company in the time period T.

2|Preliminary Results

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse U to the unit interval $[0, 1]$. **(Fuzzy Set)** A fuzzy set A in a universe of discourse U is defined as the following set of pairs: $A = \{(x, \mu_A(x)); x \in U\}$, where $\mu_A : U \longrightarrow [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in U$ in the fuzzy set A. These membership grades are often represented by real numbers lying in $[0, 1]$.

Let μ_A be the membership function of a fuzzy set A over U. The subset $supp(A) = \{x \in U \mid \mu_A(x) > 0\}$ is called support of A. We now recall that a subset $C \subseteq \mathbb{R}^n$ is convex if $(1-t)a + tb \in C$ for every $a, b \in C$ and all $t \in [0,1]$. A fuzzy set A is convex if $supp(A)$ is convex.

The subset $core(A) = \{x \in U \mid \mu_A(x) = 1\}$ is called core of A. If $core(A) \neq \emptyset$ then we say that A is normal.

(Fuzzy Number) A Fuzzy number A is a fuzzy set on the real line $\mathbb{R}$, must satisfy the following conditions.

(1) $\mu_A(x)$ is piecewise continuous.

(2) There exists at least one $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$.

(3) A must be normal and convex.

If $\alpha \in [0,1]$, the we write α-cut Some of the fuzzy numbers most known are triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy number, heptagonal fuzzy numbers, diamond fuzzy number, and pyramid fuzzy numbers. In our paper we will only use triangular fuzzy numbers.

A Triangular Fuzzy Number A is defined as $A(a,c,b)$, where a, c and b are real numbers and its membership function is given below.

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x < a, \\ \frac{x-a}{c-a} & \text{if} \quad a \leq x \leq c, \\ \frac{b-x}{b-c} & \text{if} \quad c \leq x \leq b, \\ 0 & \text{if} \quad x > b. \end{cases}$$

We can see that for each α-cut there is a unique interval $[a_\alpha, b_\alpha]$, where

$$a_\alpha = a + \alpha(c-a) \text{ and } b_\alpha = b - \alpha(b-c).$$

Fuzzy Arithmetic

Let A and B be two fuzzy numbers defined by $A(a', c', b')$ and $B(a'', c'', b'')$ with membership functions μ_A and μ_B, respectively. We have

- Addition: $(A+B)(a'+a'', b'+b'')$.
- Subtraction:$(A-B)(a'-a'', b'-b'')$.
- Multiplication:$(A \times B)(\min\{a'a'', a'b'', b'a'', b'b''\}, \max\{a'a'', a'b'', b'a'', b'b''\})$.
- Division:$(\frac{A}{B})(\min\{\frac{a'}{a''}, \frac{a'}{b''}, \frac{b'}{a''}, \frac{b'}{b''}\}, \max\{\frac{a'}{a''}, \frac{a'}{b''}, \frac{b'}{a''}, \frac{b'}{b''}\})$.
- Scalar multiplication: let $k \in \mathbb{R}, k \neq 0$.

$$kA = \begin{cases} A(ka', kb') & \text{if } k > 0 \\ A(kb', ka') & \text{if } k < 0. \end{cases}$$

3|Probabilities of the Insolvency and Inefficiency Risks

It is well known that for decision-making, prior to the beginning of a period of time T, by the managers of a service producing company, it is very useful to know previously which are the respective probabilities of the insolvency and inefficiency risks to which the company is exposed in the period T.

We claim that for to be able to associate a probability to the risks of insolvency and inefficiency, respectively, first of all, it is necessary to define the functions of sales and costs of the company. These are vital elements for develop any mathematical model with the intention of measuring in to some extent the risk of insolvency and the efficiency in period T.

The sales and costs functions must conform to the particular characteristics of the company in question. Therefore, taking into account the general characteristics of the companies of this type, we consider that these functions can be defined through linear functions of several variable. We will now show some general ideas about the definitions of these functions taking as a premise that each of them involves the same variables.

We denote the sales and costs totals by S and C, respectively. Then the respective functions of sales and costs could be defined as follows:

$$F_s(X_1, X_2, \ldots, X_t) = \sum_{\beta=1}^{t} P_\beta^s X_\beta, \tag{1}$$

where P_β^s is the sale price of the service X_β, and

$$F_c(X_1, X_2, \ldots, X_t) = \sum_{\beta=1}^{t} P_\beta^c X_\beta, \tag{2}$$

where P_β^c is the cost price of the service X_β.

We claim that in the definitions (1) and (2) there is a great question:

How to assign the respective prices of sales and costs?

Implicit in the answer to this question is the first application of the fuzzy sets in our paper. We can also see that the application of some elements of the mathematical simulation.

Let $T_1, \ldots, T_n$ be a list of n periods of time statistically select, and let P_β^{is} and P_β^{ic} be the respective prices of sales and costs of the service X_β in the the period T_i. We will now compute the confidence intervals for the lists $P_\beta^{1s}, \ldots, P_\beta^{ns}$ and $P_\beta^{1c}, \ldots, P_\beta^{nc}$. We denote these confidence intervals by $CI_{P_\beta^s} = [p'_{s\beta}, p''_{s\beta}]$ and $CI_{P_\beta^c} = [p'_{c\beta}, p''_{c\beta}]$, respectively. We can now to define the fuzzy triangular numbers associated with P_β^s and P_β^c, respectively. Thus we have:

$$P_\beta^s(P_\beta^{s\prime}, \bar{P}_\beta^s, P_\beta^{s\prime\prime}),$$

where $\bar{P}_\beta^s = \frac{\sum_{i=1}^n P_\beta^{is}}{n}$ with membership function $\mu_{P_\beta^s}$, and

$$P_\beta^c(P_\beta^{c\prime}, \bar{P}_\beta^c, P_\beta^{c\prime\prime}),$$

where $\bar{P}_\beta^c = \frac{\sum_{i=1}^n P_\beta^{ic}}{n}$ with membership function $\mu_{P_\beta^c}$.

The next step is to determine through a system of experts the corresponding α-cuts, and therefore the confidential intervals that these determine. We claim that the sales prices and costs are will taken from these intervals.

The definition of these functions plays an extraordinarily important role for make an objective of the profits forecast for the period T, and hence for the process of control and measurement systematic of the risks of insolvency and inefficiency of the company, in other words, for to develop the steps two and three. Furthermore, it turns out to be a very useful instrument in the matter concerning the fixation of the extremes of the interval of confidence of the fuzzy number associated with S.

Now we are going to associate a fuzzy triangular number to S. To do this we need to define a confidence interval. When defining the minimum and maximum extremes of that interval is where the sales function can be very useful.

Let S_i be the total sales corresponds to the period T_i. We will now compute a confidence interval for the lists $S_1, \ldots, S_n$. From this confidence interval $CI_S = [s', s'']$, we obtain the fuzzy triangular number associated with S, which is defined in the following way:

$$S(s', \bar{s}, s''),$$

where $\bar{s} = \frac{\sum_{i=1}^n S_i}{n}$ with membership function μ_S.

We now denote the total costs by C. We claim that $C = \sum_{l=1}^m C^l$, where C^l is cost according to the costs classification used in the company. Let $C_1^l, \ldots, C_n^l$ be a complete list, where each C_i^l represents the total cost C^l in the time period T_i. Now we compute a confidence interval for the lists $C_1^l, \ldots, C_n^l$. We denote this confidence interval by $CI_{C^l} = [c^{l\prime}, c^{l\prime\prime}]$. Thus, the fuzzy triangular number associated with C^l is defined in the following way:

$$C^l(c^{l\prime}, \bar{c^l}, c^{l\prime\prime}),$$

where $\bar{c^l} = \frac{\sum_{i=1}^{n} C_i^l}{n}$ with membership function μ_{C^l}.

We may then assert from all this that the fuzzy triangular number associated with C is defined in the following way:

$$C(c', \bar{c}, c'') = \sum_{l=1}^{m} C^l(c^{l\prime}, \bar{c^l}, c^{l\prime\prime}),$$

where $\bar{c} = \frac{\sum_{l=1}^{m} \bar{c^l}}{m}$ with membership function μ_C.

We denote the total profits by P. Let $P_1, \ldots, P_n$ be a complete list, where each P_i represents the total profits of the time period T_i. We may then assert that the fuzzy triangular number associated with P can be defined as follows:

$$P(p', \bar{p}, p'') = S(s', \bar{s}, s'') - C(c', \bar{c}, c''),$$

where $\bar{p} = \frac{\sum_{i=1}^{n} P_i}{n}$ with membership functions μ_P.

Probabilities of Insolvency and Inefficiency Risks

We first need to obtain the sales forecast for the period T. We will denote the sales forecast for the period T by S^T. We, in fact, claim that the sales forecast S_T can be realized maximizing the objective function (1) subject to a set of r constraints given by an equations system of form

$$AX = B. \tag{3}$$

We can assert that the constraint give by

$$\sum_{\beta=1}^{t} P_\beta^s X_\beta - X_s = \bar{s},$$

where X_s is a **slack variable**, it should always be part of the equations system (3). We may then assert that by applying the simplex method we can obtain S_T.

We claim that together with S_T, a forecast is made of the total costs C_T for the period T, in order to computer the expected profits P_T for this period. Thus we compute the α-cut obtained by $\mu_S(S_T)$. We denote this α-cut by α_{sf}. We now compute the confidence interval

$$C(c', \bar{c}, c'')(\alpha_{sf}) = [c'_{\alpha_{sf}}, \bar{c}, c''_{\alpha_{sf}}].$$

Thus it is now up to us to minimize the function (2) subject to the following restrictions:

$$\sum_{\beta=1}^{t} P_\beta^s X_\beta = S_T$$

$$\sum_{\beta=1}^{t} P_\beta^c X_\beta + X_c = c''_{\alpha_{sf}}$$

$$\sum_{\beta=1}^{t} P_\beta^p X_\beta - X_p = \bar{p},$$

where X_c and X_p are surplus and slack variables respectively, and $P_\beta^p = P_\beta^s - P_\beta^c$.

Thus applying the simplex method, we can obtain the forecast of total costs C_T for the period of time T. We can see from all this that the profits forecast P_T for the period T can be obtained as follows $S_T - C_T$.

We will now assign to the inefficiency risk a probability. We claim that $P_T \geq \bar{p}$, which means that P_T always belongs to the interval P_T. Thus the probability of risk of inefficiency is defined by the formula $P_{inf}(x) = \mu_P(P_T)$. In such a situation, we can say that the probability of inefficiency risk for the time period T is $100\mu_P(P_T)\%$.

We now need other important informations for to forecast the probability of insolvency risk of the company in the period of time T.

We will assume that each time period T_i is subdivided into z equal instants of time t_{ij} $(1 \leq j \leq z)$. Let us denote the minimum instant of time of the period T_i in which sales actual totals $S^T_{t_{ij}}$ and the actual total cost $C_{t_{ij}}$ satisfy the inequality $S_{T_i} - C_{T_i} \geq 0$ by t^0_{ij}. Let $t^0_{1j}, t^0_{2j}, \ldots, t^0_{nj}$ $(1 \leq j \leq z)$ be a complete list of all these instants of time in each T_i. Let us now define the fuzzy triangular number

$$t(t', \bar{t}, t''),$$

where $\bar{t} = \frac{\sum_{i=1}^{n} t^0_{ij}}{n}$ with membership function μ_t.

Since $\mu_P(\bar{p}) = \mu_t(\bar{t}) = 1$, we can then consider the set of points

$$(P_1, t^0_{1j}); (P_2, t^0_{2j}); \ldots; (P_n, t^0_{nj}); (\bar{p}, \bar{t}), \tag{4}$$

where P_i denote the total profits of the period T_i.

We can rewrite (4) as a set of the following form:

$$(x_0, y_0); (x_1, y_1); \ldots; (x_n, y_n). \tag{5}$$

Applying the Lagrange Interpolation Method to the set (5) of $n + 1$ points, we obtain a polynomial function $t_f : [t', t''] \longrightarrow \mathbb{R}$ given by

$$t_f(x) = \sum_{i=0}^{n} y_i l_i(x), \tag{6}$$

where $l_i(x) = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$, and $grad(t_f) = n$. The function (6) is called **solvency time**.

We compute $t^0_T = t_f(P_T)$ and the confidence interval

$$t(t', \bar{t}, t'')(\mu_P(P_T) = [t'_{\mu_P(P_T)}, \bar{t}, t''_{\mu_P(P_T)}].$$

We can deduce from $\bar{p} \leq P_T \leq p''_{\alpha_{sf}}$ that $t^0_T \in [t'_{\mu_P(P_T)}, \bar{t}]$. We may then assert that the probability of insolvency risk can be defined by the formula $P_{ins}(x) = \mu_t(t^0_T)$.

In this case, we can that the risk probability of insolvency for the time period T is $100\mu_t(t^0_T)\%$.

Control of the Insolvency and Inefficiency Risks

In this section, we develop the second step, in other words, we will show a proposal to systematically control the risk of insolvency, and hence of inefficiency, during the course of the period of time T.

First we will present an algorithm through which is possible to determine the instant of time in which the sales rate exceeds the rate of expenses, i.e. the moment in which the generating profits for the company.

We denote the total sales obtained by the company in the instant of time t_j by S_{t_j}, and let C_{t_j} be the total cost in the instant of time t_j. Let us now denote the quotients $\frac{S_{t_j}}{t_j}$ and $\frac{C_{t_j}}{t_j}$ by V^S_j and V^C_j, respectively. These quotients are called sales rhythm and costs rhythm, respectively.

The difference $D_{t_j} = V^S_j - V^C_j$ is called rhythm differential. We claim that if the rhythm differential is negative the company runs the risk of being insolvent, and hence inefficient. Let us denote the minimum value of t_j for which $D_{t_j} \geq 0$ by t^0_j. The fact $D_{t^0_j} \geq 0$ could be interpreted as a fusion of insolvency and inefficiency risks. At this moment we can only assert that if the difference $t^0_T - t^0_j \geq 0$ then the risks of insolvency and inefficiency have been reduced.

Now, if $t^0_j < \bar{p}$ then we may assert that the probability of insolvency and inefficiency of the company is $100\mu_p(t^0_j)\%$. The inequality $t^0_j \geq \bar{p}$ means **maximum probability of being inefficient in period** T.

We claim that to achieve maximum efficiency in the period of time T it is necessary to systematically maintain a strict cost control. We then compute $\mu_S(S_j) = \alpha_j$ for each j. Furthermore, we determine the confidence intervals

$$[c^{l\prime}_{\alpha_j}, c^{l\prime\prime}_{\alpha_j}]$$

for each l.

We now denote the cost C^l in the instant of time t_j by C_j^l. If $S_j \geq \bar{s}$ $(S_j < \bar{s})$ and $C_j^l \in [\bar{c^l}, c_{\alpha_j}^{l\prime\prime}]$ $C_j^l \in [c_{\alpha_j}^{l\prime}, \bar{c^l})$, for all $l, 1 \ \leq l \leq m$, then we can say that the rhythm differential is stable. Otherwise the rhythm differential is is unstable.

We can say from this that for the company to be solvent and hence efficient, it is necessary that the rhythm differential be stable during the entire period T. We can then see that a disturbance in the rhythm differential in any time could even cause the company to be insolvent.

Let us denote the minimum instant of t_j for which $P_{t_j} = S_{t_j} - C_{t_j} \geq \bar{p}$ by t_j^e. We can immediately deduce that if $j < z$ then we cannot conclude that the company is efficient, since until the last instant of time t_z it must be noted that the differential of rhythm is stable. This means that only an analysis exhaustive in this aspect at the end of the period T can provide that information.

4|Control of the Insolvency and Inefficiency Risks

In this section, we develop the second step, in other words, we will show a proposal to systematically control the risk of insolvency, and hence of inefficiency, during the course of the period of time T.

First we will present an algorithm through which is possible to determine the instant of time in which the sales rate exceeds the rate of expenses, i.e. the moment in which the generating profits for the company.

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The difference $D_{t_j} = V_j^S - V_j^C$ is called rhythm differential. We claim that if the rhythm differential is negative the company runs the risk of being insolvent, and hence inefficient. Let us denote the minimum value of t_j for which $D_{t_j} \geq 0$ by t_j^0. The fact $D_{t_j^0} \geq 0$ could be interpreted as a fusion of insolvency and inefficiency risks. At this moment we can only assert that if the difference $t_T^0 - t_j^0 \geq 0$ then the risks of insolvency and inefficiency have been reduced.

Now, if $t_j^0 < \bar{p}$ then we may assert that the probability of insolvency and inefficiency of the company is $100\mu_p(t_j^0)\%$. The inequality $t_j^0 \geq \bar{p}$ means **maximum probability of being inefficient in period** T.

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$$[c_{\alpha_j}^{l\prime}, c_{\alpha_j}^{l\prime\prime}]$$

for each l.

We now denote the cost C^l in the instant of time t_j by C_j^l. If $S_j \geq \bar{s}$ $(S_j < \bar{s})$ and $C_j^l \in [\bar{c^l}, c_{\alpha_j}^{l\prime\prime}]$ $C_j^l \in [c_{\alpha_j}^{l\prime}, \bar{c^l})$, for all $l, 1 \ \leq l \leq m$, then we can say that the rhythm differential is stable. Otherwise the rhythm differential is is unstable.

We can say from this that for the company to be solvent and hence efficient, it is necessary that the rhythm differential be stable during the entire period T. We can then see that a disturbance in the rhythm differential in any time could even cause the company to be insolvent.

Let us denote the minimum instant of t_j for which $P_{t_j} = S_{t_j} - C_{t_j} \geq \bar{p}$ by t_j^e. We can immediately deduce that if $j < z$ then we cannot conclude that the company is efficient, since until the last instant of time t_z it must be noted that the differential of rhythm is stable. This means that only an analysis exhaustive in this aspect at the end of the period T can provide that information.

5|Analysis of the Financial Efficiency

It is clear that measuring the efficiency of the company can only be done once the time period T has ended. The measurement of the efficiency of the company in the time period T is divided into two levels: minimum level of efficiency and maximum level of efficiency.

Minimum Level of Efficiency: The measurement is made without taking into account the sales forecast for the period T.

Maximum Level of Efficiency: The measurement is made keeping in mind the sales forecast for the period T.

Let S^T, C^T and P^T be the total sales, total costs and net profit obtained in the period T, respectively.

Minimum Efficiency

First of all, we must check if the inequality $\bar{s} \leq S^T$ is true. We, in fact, claim that if $\bar{s} \lneq S^T$, then we can automatically assert that the company did not reach the minimum level of efficiency in the period T.

We assume that $\bar{s} \leq S^T$ is true. In this situation, let us write α^T for $\mu_S(S^T)$.

We compute the confidence intervals $[c'_{\alpha^T}, c''_{\alpha^T}]$ and $[p'_{\alpha^T}, p''_{\alpha^T}]$.

We can assert that the company has reached minimum efficiency in time period T if and only if it meets the following indicators:

(1) $\bar{s} \leq S^T$.

(2) $C^T \leq c''_{\alpha^T}$.

(3) $\bar{p} \leq P^T$.

If the company fails to comply with some of these indicators, then the company immediately declares itself inefficient in period T.

We can assert that the company has reached maximum efficiency in time period T if and only if it meets the following indicators:

(1) $S_T \leq S^T$.

(2) $C^T \leq C_T$.

(3) $P_T \leq P^T$.

6|Conclusion

For a company that produces services, the risks of insolvency and inefficiency that the company runs during a period of time T are phenomena submerged in the deepest uncertainty which are inextricably linked. The Fuzzy arithmetic turns out to be an essential tool in concerning the mathematical moderation of instruments intended for the measurement and control of these complex phenomena. In this article a mathematical model is shown that can be used in three moments of great importance in this context, which can be summarized as follows: forecasting the risk of insolvency and inefficiency, control of the risk of insolvency and inefficiency and measurement of the risk of insolvency and inefficiency. We can conclude by stating that the application of this mathematical model promotes a rigorous control of the company accounting and finances.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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